HIGGS MECHANISM AND SPONTANEOUS SYMM. BREAKING

In the SM, particles acquire mass through their interaction with the Higgs field. (a scalar field).

To illustrate the mechanism:

1) Show how mass terms arise for broken symm. in the case of a simple symmetry $\varphi \to -\varphi$
2) A global $U(1)$ symmetry
3) Show that masses of gauge bosons are generated from broken local $U(1)$ symmetry
4) The SM Higgs mechanism is developed by breaking $SU(2)_L \times U(1)_Y$

   Detailed calculations beyond the scope of this lecture → see PHY 452 at Master level

Start with 1): A world made just by scalar particles

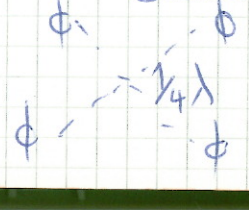
Consider a scalar field ϕ and the potential:

$$V(\phi) = \frac{1}{2}\mu^2\phi^2 + \frac{1}{4}\lambda\phi^4 \quad , \mu, \nu \text{ real constants}, \lambda > 0$$

The Lagrangian is given by

$$\mathcal{L} = \frac{1}{2}(\partial_\mu\phi)(\partial^\mu\phi) - V(\phi) =$$

$$= \underbrace{\frac{1}{2}(\partial_\mu\phi)(\partial^\mu\phi)}_{\text{kinetic energy}} \underbrace{- \frac{1}{2}\mu^2\phi^2}_{\text{mass term}} \underbrace{- \frac{1}{4}\lambda\phi^4}_{\text{self-interaction of the scalar field}}$$



$\mathcal{L}$ is invariant under the symmetry $\phi \to -\phi$.

To find the spectrum of particles:

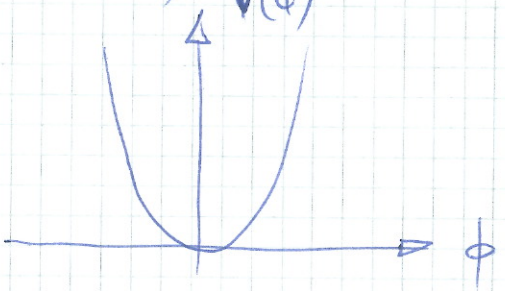
- Find the minimum of the potential (ground state of the system) } VACUUM
- Expand the fields around their value at the minimum to find the excitations } PARTICLES

(This is the standard procedure to handle perturbations in QM)

i) Case in which $\mu^2 > 0$:

Describes a scalar field with mass μ (as we have seen previously)

For $\mu^2 > 0$



$V(0) = 0$

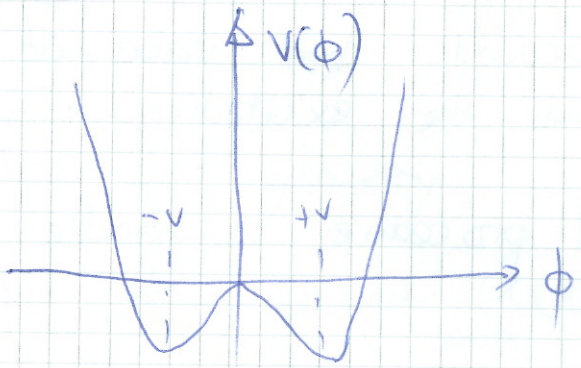
The ground state corresponds to $\phi = 0$.

Note that in this case the ground state obeys the reflection symmetry $\phi \to -\phi$.

ii) Case in which $\mu^2 < 0$

Minimum of the potential:

$$\frac{\partial V}{\partial \phi} = \phi(\mu^2 + \lambda\phi^2) = 0 \quad \Rightarrow \phi = \pm\sqrt{\frac{-\mu^2}{\lambda}} \equiv \pm v$$



Note:
- $\phi = 0$ does not correspond to the minimum of the potential
- The minimum does not obey the reflection symmetry.

Since ϕ takes the value v in the ground state,

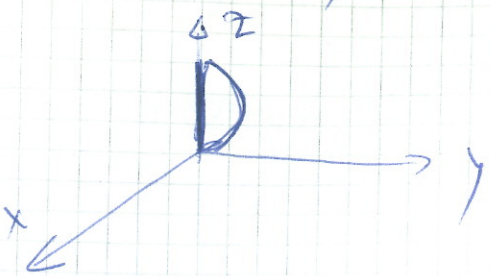
> v is called the vacuum expectation value

The actual vacuum state will be either $+v$ or $-v$.

$\Rightarrow$ The choice of the vacuum state breaks the symmetry

$\Rightarrow$ THIS IS KNOWN AS SSB (Spontaneous Symmetry Breaking)

Ex: A needle, when compressed, will bend in an arbitrary direction



⇒ The original cylindrical symmetry of the system is broken

(even though the direction of the applied force is symm)

To get the particle spectrum, we must do a perturbative expansion around the MINIMUM,

$\phi = +v$ or $\phi = -v$.

(The physical conclusions cannot depend on this choice, remember $\mathcal{L}$ is invariant under $\phi \to -\phi$)

We write $\phi(x) = v + \eta(x)$

v: const

$\eta(x)$: quantum fluctuations around the MINIMUM

Substitute into the Lagrangian:

$$\mathcal{L}' = \frac{1}{2}(\partial_\mu \eta)(\partial^\mu \eta) - V(\eta) =$$

$$= \frac{1}{2}(\partial_\mu \eta)(\partial^\mu \eta) - \frac{1}{2}\mu^2(v+\eta)^2 - \frac{1}{4}\lambda^4(v+\eta)^4$$

With $\mu^2 = -\lambda v^2$

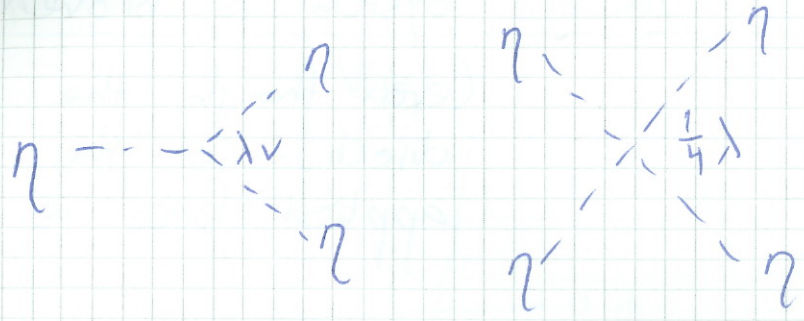
$$\mathcal{L}' = \underbrace{\frac{1}{2}(\partial_\mu \eta)(\partial^\mu \eta)}_{\text{kin. term}} \underbrace{- \lambda v^2 \eta^2}_{\text{mass term}} \underbrace{- \lambda v \eta^3 - \frac{1}{4}\lambda \eta^4}_{\text{interaction terms}} + \underbrace{\frac{1}{4}\lambda v^4}_{\text{const}}$$

$\mathcal{L}$ no phys

Mass term $\lambda v^2 > 0 \Rightarrow$ mass term has correct sign.

$\mathcal{L}'$ describes a particle with mass $m_\eta = \sqrt{2\lambda v^2} = \sqrt{-2\mu^2} \in \mathbb{R}$

The 3rd and 4th term $(\lambda v \eta^3, \eta^4)$ represent interactions of the form



Self-interactions with strength λv and $\frac{1}{4}\lambda$, where λ is a free parameter.

$$\mathcal{L}' = \frac{1}{2}(\partial_\mu \eta)(\partial^\mu \eta) - \frac{1}{2} m_\eta^2 \eta^2 - V(\eta)$$

where $V(\eta) = \lambda v \eta^3 + \frac{1}{4}\lambda \eta^4$

Note: $\mathcal{L}$ and $\mathcal{L}'$ describe the same physical system (we have only changed the notation.)

To get the particle spectrum we must expand around the minimum (and not around $\phi = 0$, this would be unstable)

$\Rightarrow$ $\mathcal{L}$ and $\mathcal{L}'$ describe a massive scalar particle!

Summary so far:

This was a simple example of SSB:

- The original $\mathcal{L}$ is invariant under $\phi \to -\phi$
- The reformulated $\mathcal{L}'$ is not, not even in η.
  ⇒ The symmetry has been broken.
- The reason for this is that the ground state, or vacuum, does not share the symmetry of the Lagrangian. Therefore, the choice of the ground state breaks the symmetry.
- In the perturbative expansion the mass is "generated" or "revealed".

2) A complex scalar field that obeys and a Lagrangian U(1) symmetry

We now apply the idea of SSB to a scalar field

$$\phi = \frac{1}{\sqrt{2}}(\phi_1 + i\phi_2)$$

and the Lagrangian

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^*(\partial^\mu \phi) \underbrace{- \mu^2 \phi^* \phi - \lambda(\phi^* \phi)^2}_{-V(\phi)}$$

The Lagrangian is invariant under a transformation $\phi \to \phi' = e^{i\alpha}\phi$, since $\phi'^* \phi' = \phi^* \phi$

and thus possesses a global U(1) symmetry.

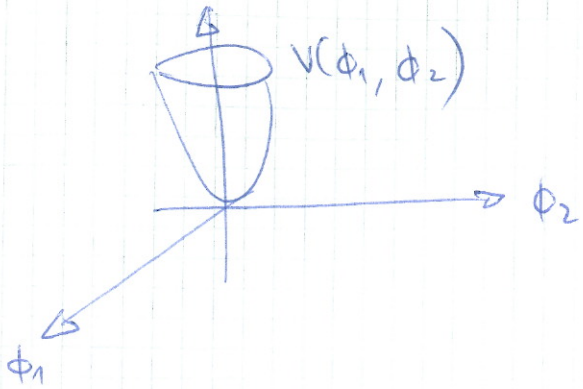
The shape of the potential depends again on the sign of μ^2.

Express $\mathcal{L}$ in terms of the real fields ϕ_1 and ϕ_2.

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi_1)^2 + \frac{1}{2}(\partial_\mu \phi_2)^2 \underbrace{- \frac{1}{2}\mu^2(\phi_1^2 + \phi_2^2) - \frac{1}{4}\lambda(\phi_1^2 + \phi_2^2)^2}_{V(\phi_1, \phi_2)}$$

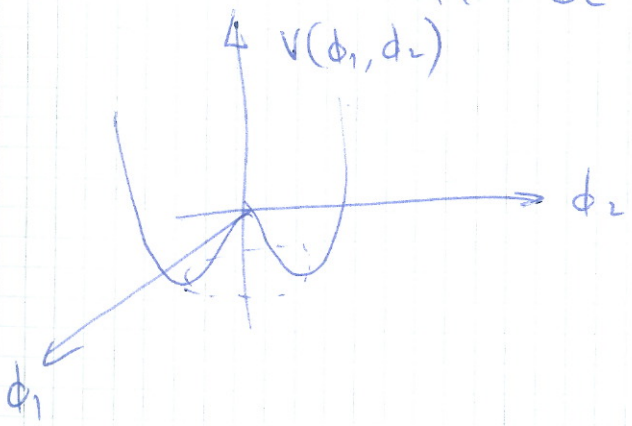
i) For $\mu^2 > 0$

The minimum of the potential occurs when both fields are zero $\phi_1 = \phi_2 = 0$



For $\mu^2 < 0$: The minimum of $V(\phi_1, \phi_2)$ occurs in the (ϕ_1, ϕ_2) - plane in a circle with radius

$$\phi_1^2 + \phi_2^2 = -\frac{\mu^2}{\lambda} = v^2$$



"Mexican hat potential"

The physical vacuum state corresponds to a particular point on this circle.

Chose the vacuum choice in the real direction and expand around the point $(\phi_1, \phi_2) = (v, 0)$:

$$\phi(x) = \frac{1}{\sqrt{2}} \left(\overbrace{v + \eta(x)}^{\phi_1} + \overbrace{i\rho(x)}^{i\phi_2} \right)$$

η, ρ : real functions

Express the Lagrangian in terms of η, ρ:

$$\mathcal{L}' = \overbrace{\tfrac{1}{2}(\partial_\mu \rho)^2 + \tfrac{1}{2}(\partial_\mu \eta)^2}^{\text{kinetic terms}} + \overbrace{\mu^2 \eta^2}^{\text{mass term}}$$

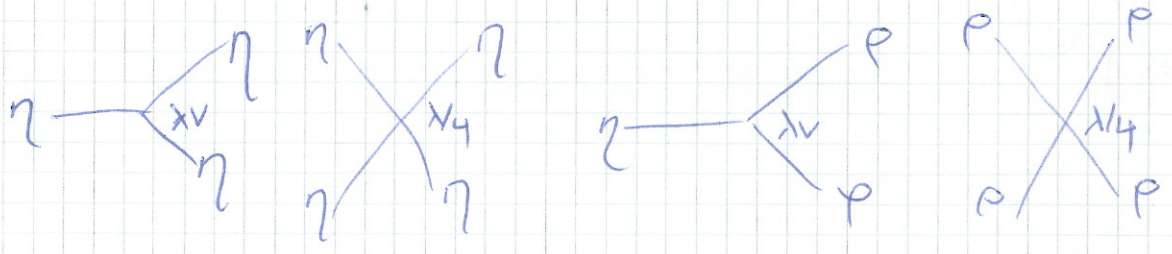
$$\underbrace{- \lambda v (\eta \rho^2 + \eta^3) - \tfrac{\lambda}{2}\eta^2\rho^2 - \tfrac{\lambda}{4}\eta^4 - \tfrac{\lambda}{4}\rho^4}_{\text{interaction terms}} + \text{const}$$

$$= \tfrac{1}{2}(\partial_\mu \rho)^2 + \tfrac{1}{2}(\partial_\mu \eta)^2 - \tfrac{1}{2} m_\eta^2 \eta^2 - V_{int}(\eta, \rho)$$

$$m_\eta = \sqrt{2\lambda v^2} = \sqrt{-2\mu^2} > 0$$

Note:

i) Get kinematic terms for η and ρ
ii) Get mass term with correct sign for η
iii) Mass term for ρ cancels → field ρ is massless.
iv) Interaction terms of the form



⇒ $\mathcal{L}'$ represents a massive scalar field η and a massless scalar field ρ (also known as a Goldstone boson)

3) Our closest approach to the SM Higgs mechanism:

A local U(1) invariance

$$\phi(x) \to \phi'(x) = e^{i\alpha(x)}\,\phi(x)$$

and the potential $V(\phi) = \mu^2\phi^2 + \lambda\phi^4$

To get the required local U(1) gauge invariance, replace the derivatives ∂_μ in $\mathcal{L}$ with the covariant derivatives D_μ.

$D_\mu = \partial_\mu - ieA_\mu$ with the gauge field

$$A_\mu \to A_\mu' = A_\mu + \frac{1}{e}\partial_\mu\alpha$$

The resulting Lagrangian is

$$\mathcal{L} = (D_\mu\phi)^*(D^\mu\phi) - \mu^2\phi^*\phi - \lambda(\phi^*\phi)^2 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

i) $\mu^2 > 0$: $\mathcal{L}$ describes the interaction of a charged, scalar particle with mass μ with the e.m. field A_μ. There is no mass term for A_μ.

b) $\mu^2 < 0$: We want to generate masses via SSB.

Replace $\phi(x) = \frac{1}{\sqrt{2}}(v + \eta(x) + i\rho(x))$ in $\mathcal{L}$:

$$\mathcal{L}' = \overbrace{\frac{1}{2}(\partial_\mu\eta)(\partial^\mu\eta) - \lambda v^2\eta^2}^{\text{kin + mass term for } \eta} + \overbrace{\frac{1}{2}(\partial_\mu\rho)(\partial^\mu\rho)}^{\text{kin term for } \rho}$$

$$\overbrace{-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}e^2v^2A_\mu A^\mu}^{\text{massive gauge field}} - V_{int}(\eta,\rho,A^\mu)$$

$$+ evA_\mu(\partial^\mu\rho)$$

Thus we get:

i) a massive scalar particle η with $m_\eta = \sqrt{2\lambda v^2}$

ii) a massless Goldstone boson ρ

iii) a massive vector boson A_μ with $m_A = ev$

has a mass if $v \neq 0$, i.e. symm. is broken.

However there are two problems with $\mathcal{L}'$:

1) The original $\mathcal{L}$ has 4 degrees of freedom:
   2 real scalars ϕ_1, ϕ_2
   2 polarization states of the mass less field A_μ

   The new $\mathcal{L}'$ has an additional degree of freedom: longitudinal polarization of massive A_μ

2) Interaction term of spin-0 (Goldstone field) and spin-1 (A) field

   A ~~~~ ρ $\quad (ev A_\mu \partial^\mu \rho)$

Solution: Not all fields in $\mathcal{L}'$ are physical
↳ can find a gauge transformation to eliminate them.

Use the fact that to first order

$$\phi = \frac{1}{\sqrt{2}}(v + \eta + i\rho) \approx \frac{1}{\sqrt{2}}(v+\eta)e^{i\rho/v}$$

This is suggestive of another transformation

$$\phi(x) \to \phi'(x) = \frac{1}{\sqrt{2}}(v + h(x))\, e^{i\theta(x)/v}$$

$$A_\mu \to A_\mu + \frac{1}{ev}\partial_\mu \theta$$

We chose the phase θ such that h is real.

The Lagrangian can now be written as:

$$\mathcal{L}' = \frac{1}{2}(\partial_\mu h)^2 - \lambda v^2 h^2 + \frac{1}{2} e^2 v^2 A_\mu^2 - \lambda v h^3 - \frac{1}{4}\lambda h^4 + \frac{1}{2} e^2 A_\mu^2 h^2 + v e^2 A_\mu h - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

$\Rightarrow$ The Goldstone boson no longer appears in $\mathcal{L}'$! has been "eaten".

- The apparent new freedom corresponds to the freedom of a gauge transformation.
- The physical predictions of the theory are unchanged by this gauge transformation
- The gauge in which the Higgs boson is real is known as the Unitary gauge.

Left with

i) a real Higgs field h with mass $M_h = \sqrt{2\lambda v^2}$

ii) a massive vector boson A_μ with 3 spin states $(J_z = +1, 0, -1)$ with $M_A = ev$

iii) various interactions.

$\Rightarrow$ Get 3 massive gauge bosons and a Goldstone boson

4) The Higgs mechanism in the SM is derived from local $SU(2)_L \times U(1)_Y$ invariance

$$\mathcal{L}_H = \left| \left(i\partial_\mu - g \frac{\vec{\tau}}{2} \cdot \vec{W}_\mu - g' \frac{Y}{2} B_\mu \right) \phi \right|^2 - V(\phi)$$

(g: isospin, $\vec{W}_\mu$: triplet, g': hypercharge, B_μ: singlet, $V(\phi)$: Higgs potential)

$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$ doublet complex fields $\quad V(\phi) = \mu^2 \phi^\dagger \phi + \lambda (\phi^\dagger \phi)^2$

↳ 4 real fields, 3 get mass through use + 1 Higgs boson

$\Rightarrow M_W = \frac{1}{2} v g$, $\frac{M_W}{M_Z} = \cos\theta_W$, $\tan\theta_W = \frac{g'}{g}$