

experimental findings
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composite Higgs models
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running coupling
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spectrum
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Large scale separation and resonances within LHC range from a prototype model for new physics

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THE UNIVERSITY
of EDINBURGH

Zürich, November 8, 2016

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based on

R. Brower, A. Hasenfratz, C. Rebbi, E. Weinberg, O.W. PRD 93 (2016) 075028

A. Hasenfratz, C. Rebbi, O.W. arXiv:1609.01401

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experimental findings

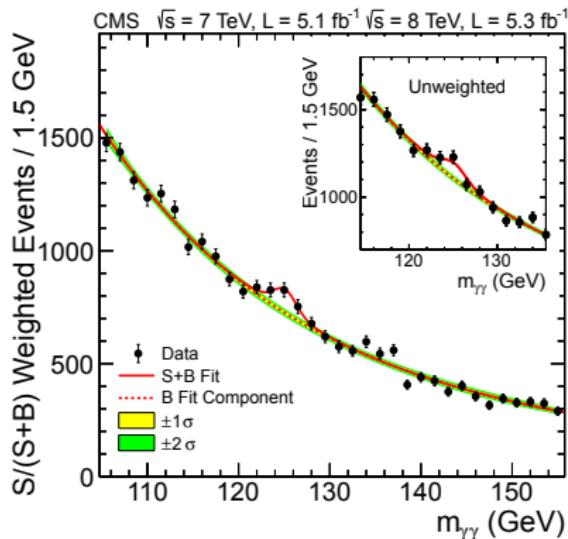
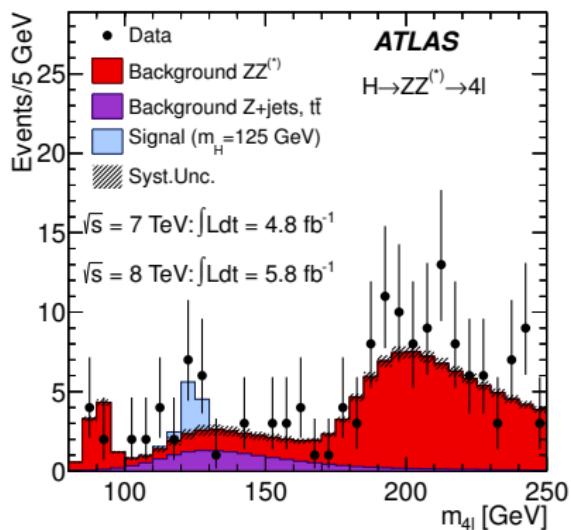
The Standard Model of Elementary Particle Physics

	generations			gauge forces	Higgs boson
quarks	I u	II c	III t	g	H
	d	s	b	γ	
leptons	ν_e	ν_μ	ν_τ	Z^0	
	e	μ	τ	W^\pm	

- ▶ Gravitation, dark matter, or dark energy not included
- ▶ Matter/antimatter asymmetry
- ▶ Is the Higgs boson a fundamental scalar?
- ▶ What happens up to $M_{\text{Planck}} \approx 1.2 \cdot 10^{19} \text{ GeV}$?

The Higgs boson

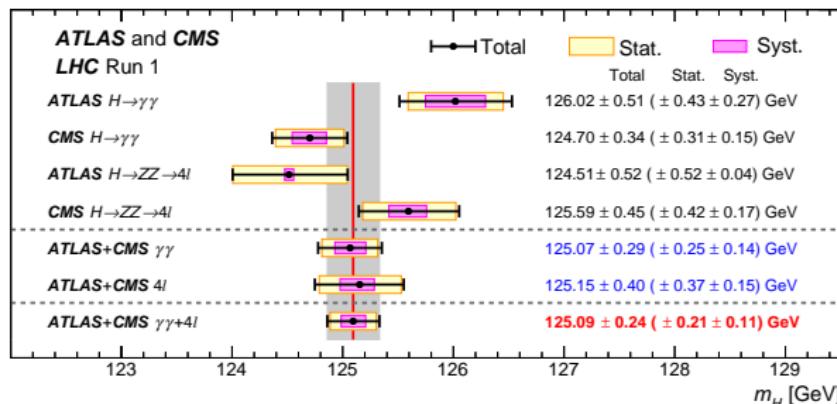
- ▶ Discovery of the Higgs boson in 2012 [Atlas PLB 716 (2012) 1] [CMS PLB 716 (2012) 30]



The Higgs boson

- Combined analysis 2015: [Atlas and CMS PRL 114 (2015) 191803]

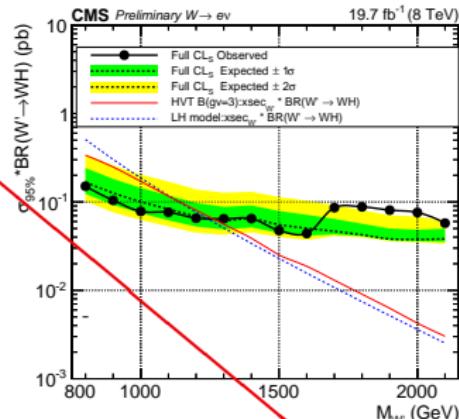
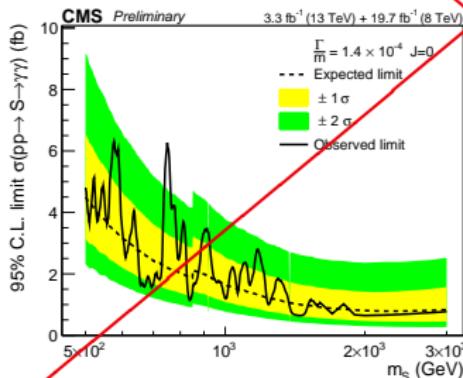
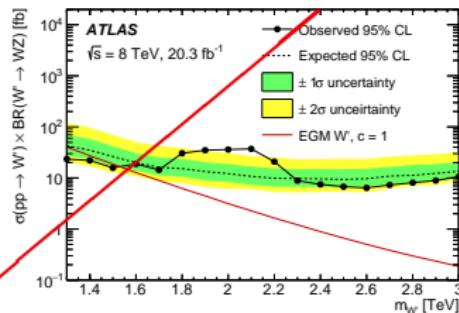
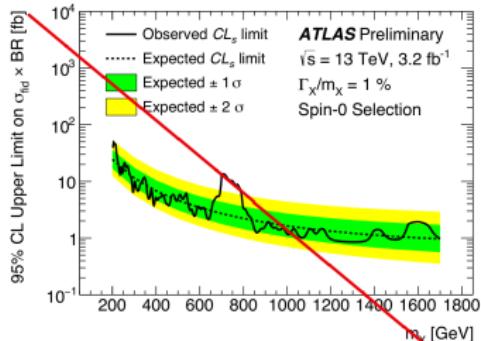
$$m_H = 125.09 \pm 0.21(\text{stat}) \pm 0.11(\text{syst}) \text{ GeV, spin 0, parity +1}$$



- We still do not know:
 - What is the Higgs boson / Standard Model UV completion?
 - What is the nature of electro-weak symmetry breaking?

Too good to be true?

ICHEP 2016: no bump at 750 GeV; no bump at 2 TeV



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Large scale separation

- ▶ Mass of the Higgs boson is 125 GeV
 - ▶ Other states must be much heavier, likely > 1.5 TeV
 - ▶ Standard Model not UV complete
- ⇒ Seek a model exhibiting a large separation of scales

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Composite Higgs models

- ▶ New, strongly interacting gauge fermion system
- ▶ Effective theory describing part of the dynamics
- ▶ Coupled to the Standard Model

$$\mathcal{L}_{UV} \rightarrow \mathcal{L}_{SD} + \mathcal{L}_{SM_0} + \mathcal{L}_{int} \rightarrow \mathcal{L}_{SM} + \dots$$

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Composite Higgs models

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Higgs-less, massless SM → “full” SM

$$\mathcal{L}_{UV} \rightarrow \mathcal{L}_{SD} + \mathcal{L}_{SM_0} + \mathcal{L}_{int} \rightarrow \mathcal{L}_{SM} + \dots$$

Composite Higgs models

- ▶ New, strongly interacting gauge fermion system
- ▶ Effective theory describing part of the dynamics
- ▶ Coupled to the Standard Model

Add new strong dynamics coupled to SM

$$\mathcal{L}_{UV} \rightarrow \mathcal{L}_{SD} + \mathcal{L}_{SM_0} + \mathcal{L}_{int} \rightarrow \mathcal{L}_{SM} + \dots$$



Full SM + states from \mathcal{L}_{SD}

This construction gives mass to:

- ▶ the SM gauge fields
- ▶ the SM fermions fields: 4-fermion interaction or partial compositeness

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Composite Higgs models

- ▶ New, strongly interacting gauge fermion system
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$$\mathcal{L}_{UV} \rightarrow \mathcal{L}_{SD} + \mathcal{L}_{SM_0} + \mathcal{L}_{int} \rightarrow \mathcal{L}_{SM} + \dots$$


Full SM + states from \mathcal{L}_{SD}

This construction gives mass to:

- ▶ the SM gauge fields
- ▶ the SM fermions fields: 4-fermion interaction or partial compositeness

Does not explain mass of \mathcal{L}_{SD} fermions and 4-fermion interactions: \mathcal{L}_{UV}

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Candidates for \mathcal{L}_{SD}

- Promising candidates are chirally broken in the IR but conformal in the UV

[Luty and Okui JHEP 09(2006)070], [Dietrich and Sannino PRD75(2007)085018],

[Vecchi arXiv:1506.00623], [Ferretti and Karateev JHEP 1403 (2014) 077], ...



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Many possibilities:

- SU(3) gauge with 4 flavors
- SU(4) with 2 diff. representations
- SU(3) gauge with 8 flavors
- SU(3) gauge with 2 sextet
- SU(2) gauge with 2 flavors

Candidates for \mathcal{L}_{SD}

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Many possibilities:

- Add fermions to push system into the conformal window
- If fermions are massive, they decouple at Λ_{IR}
- Dynamics beyond Λ_{UV} gives mass to add. fermions
- SU(3) gauge with 4 flavors
- SU(4) with 2 diff. representations
- SU(3) gauge with 8 flavors
- SU(3) gauge with 2 sextet
- SU(2) gauge with 2 flavors

Two possibilities for a composite Higgs (IR sector)

- ▶ Spontaneous breaking of **scale** symmetry: **Higgs is a dilaton**
 - Possibly light 0^{++} scalar
 - $F_\pi = \text{SM vev} \sim 246 \text{ GeV}$
 - ideal 2 massless flavors in the IR
 - closer to old technicolor ideas
- ▶ Spontaneous breaking of **flavor** symmetry: **Higgs is a pNGB**
 - Mass emerges from its interactions
 - Non-trivial vacuum alignment $F_\pi = (\text{SM vev}) / \sin(\chi) > 246 \text{ GeV}$
 - ideal 4 massless flavors in the IR
 - Vecchi: UV-complete models requiring at least two types of fermions in two different gauge group representations [[arXiv:1506.00623](#)]
 - Ferretti: Classification of models with custodial symmetry and partial compositeness [[JHEP 1606 \(2016\) 107](#)]
 - Ma and Cacciapaglia: Fundamental composite 2HDM with 4 flavors [[JHEP 03 \(2016\) 211](#)]

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Fundamental composite 2HDM with 4 flavors

[Ma and Cacciapaglia JHEP 03 (2016) 211]

- ▶ Global symmetry at low energies:

$$SU(4) \times SU(4) \text{ broken to } SU(4)_{\text{diag}}$$

- ▶ 15 pNGB transform under custodial symmetry

$$SU(2)_L \times SU(2)_R$$

$$\Rightarrow \mathbf{15}_{SU(4)_{\text{diag}}} = (2, 2) + (2, 2) + (3, 1) + (1, 3) + (1, 1)$$

- One doublet plays the role of the Higgs doublet field
- Other doublet and triplets are stable; could play role of dark matter
- ▶ Vecchi: “choose the right couplings to RH top” [Edinburgh talk]
 - ~~(2, 2) + (2, 2) + (3, 1) + (1, 3) + (1, 1)~~
 - ~~> effectively $SU(4)/Sp(4)$

Investigate specific models

- ▶ Choose N_c color and N_f flavors in a given representation
- ▶ Hopefully close to the conformal boundary but chirally broken
- ▶ Two candidates
 - SU(3) gauge with $N_f = 2$ sextet fermions (LatHC)
 - SU(3) gauge with $N_f = 8$ fundamental fermions (LatKMI, LSD)

- ▶ Certainly these two models are near the conformal boundary
but are they still chirally broken?

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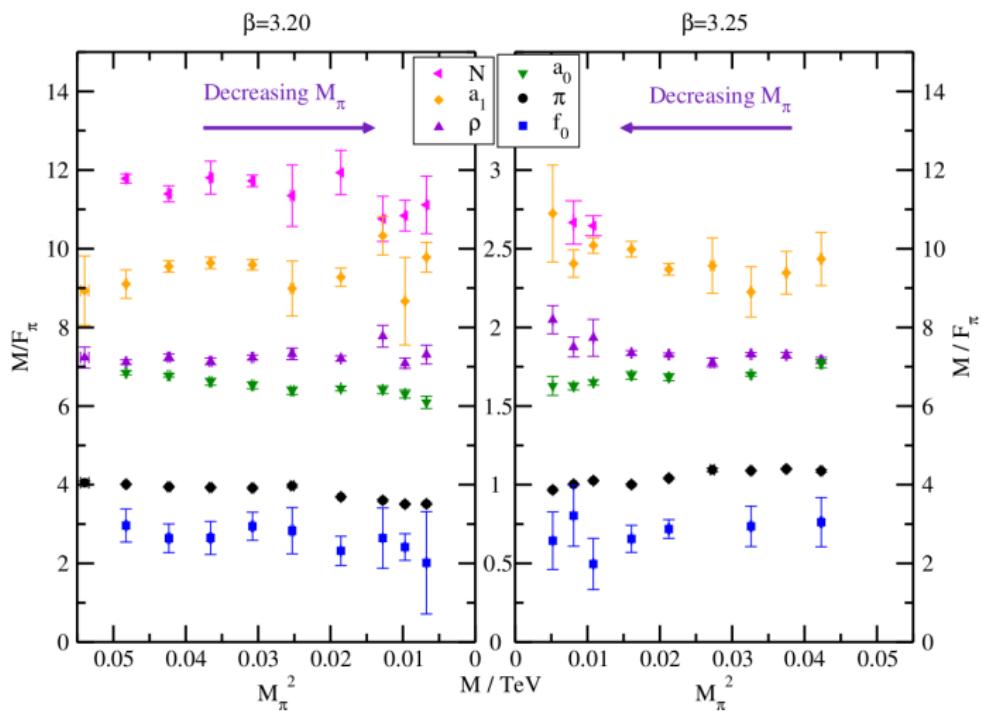
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$N_f = 2$ sextet model (LatHC) [J. Kuti, Argonne 2016]



- ▶ $M_{0^{++}} < M_\pi$
 - ▶ Very little curvature in M_π/F_π
 - ▶ But “hyperscaling does not fit”
 - ▶ β function is small
 - ▶ Baryons candidates for dark matter?
- [PRD 94 (2016) 014503]

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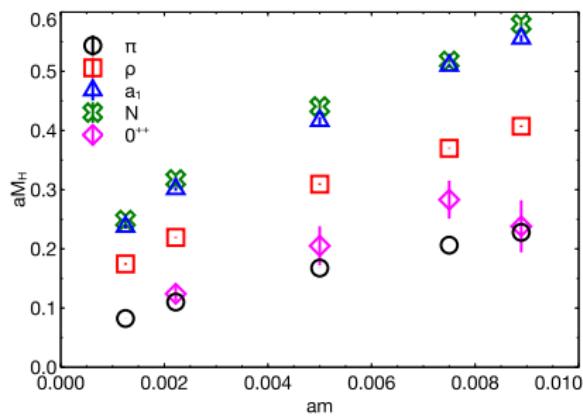
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$N_f = 8$ fundamental

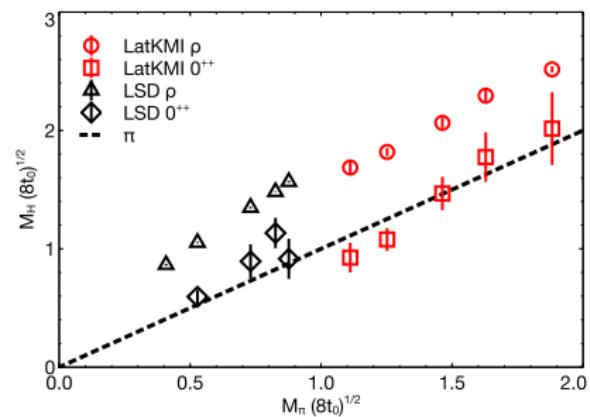
► LSD [PRD 93 (2016) 114514]

→ nHyp smeared staggered fermions



► LatKMI [PRD 87 (2013) 094511]

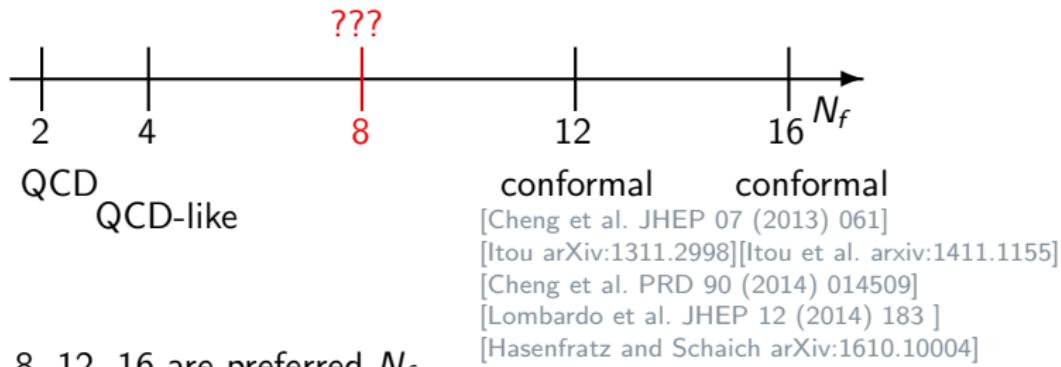
→ HISQ fermions



- Spectrum matches despite different actions ↠ small discretization errors
- Very little curvature in M_π/F_π ; cannot say whether system is chirally broken
- Pallante et al. [arXiv:1506.06396] and Iwasaki et al. [PLB 748 (2015) 289] claim it is conformal

Alternatively: build a model on a conformal fixed point

- ▶ Built in features (e.g. [Luty and Okui JHEP 09 (2006) 070])
 - Conformal in the UV (non-perturbative)
 - Chirally broken in the IR
 - Walking gauge coupling
- ▶ SU(3) gauge theories with N_f fundamental fermions
- ▶ Staggered fermions come in multiplicities of 4 (no rooting)



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4+8 model as a candidates for \mathcal{L}_{SD}

- SU(3) gauge theory with 4 light (massless) and 8 heavy fundamental flavors



- Add 8 “heavy” fundamental flavors
 - $N_f = 4 + 8 = 12$:
 - conformal dynamics
 - SU(3) gauge theory with 4 light, fundamental flavors
 - Prototype pNGB or dilaton-Higgs
-
- Ensures chiral symmetry breaking in the IR
 - Tune “walking” by changing m_h
 - Anomalous dimensions correspond to the conformal IRFP
 - ~~ One implementation; many other possibilities

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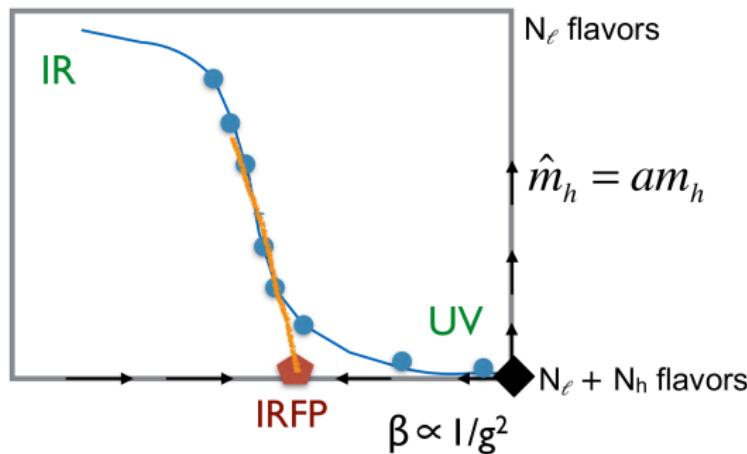
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Implementation on the lattice

- ▶ Choose N_f flavors above the conformal window
- ▶ Split the masses: $N_f = N_\ell + N_h$
 - ▶ N_h flavors are massive, we will vary $m_h \rightarrow$ decouple in the IR
 - ▶ N_ℓ flavors are massless, extrapolate $m_\ell \rightarrow 0 \Rightarrow$ chirally broken



- ▶ We have 3 parameters:
 - g irrelevant coupling
 - $m_\ell \rightarrow 0$ (chiral limit)
 - m_h : sets the scale
- allows to tune walking

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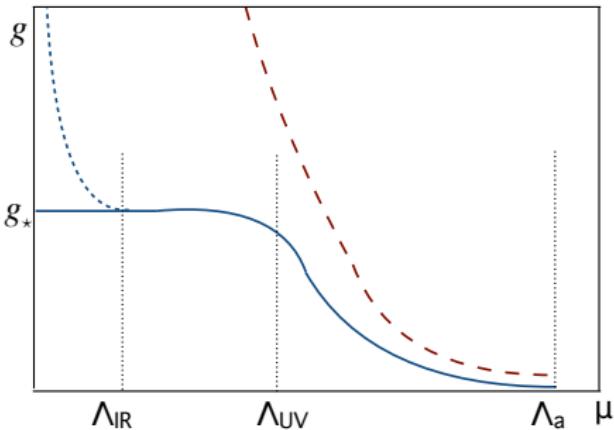
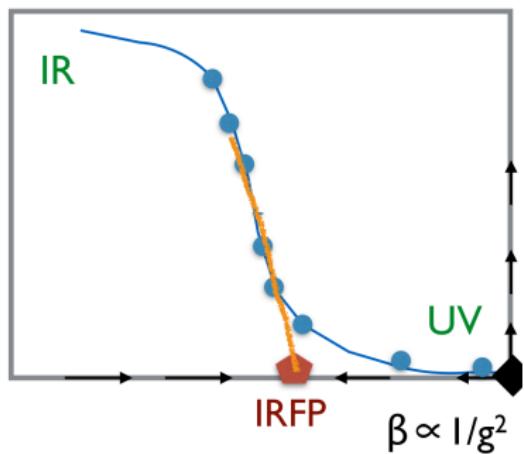
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running coupling

Running coupling in mass-split systems



- ▶ UV to cut-off: $g \sim g^*$
- ▶ Walking: m_h small, $g \sim g^*$
- ▶ IR: heavy flavors decouple;
 N_ℓ light flavors are chirally broken
- ▶ Length of walk depends on anomalous dimension

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Running coupling form gradient flow

- ▶ Gradient flow defines the renormalized coupling

[Narayanan and Neuberger JHEP 03 (2006) 064], [Lüscher JHEP 08 (2010) 071]

$$g_{GF}^2(\mu = 1/\sqrt{8t}) = t^2 \langle E(t) \rangle / \mathcal{N}$$

t : flow time; $E(t)$ energy density

- ▶ g_{GF}^2 is used for scale setting

$$g_{GF}^2(t = t_0) = 0.3/\mathcal{N} \quad (\text{"}t_0\text{-scale"})$$

- ▶ Can determine renormalized running coupling on large enough volumes and large enough flow times in the continuum limit

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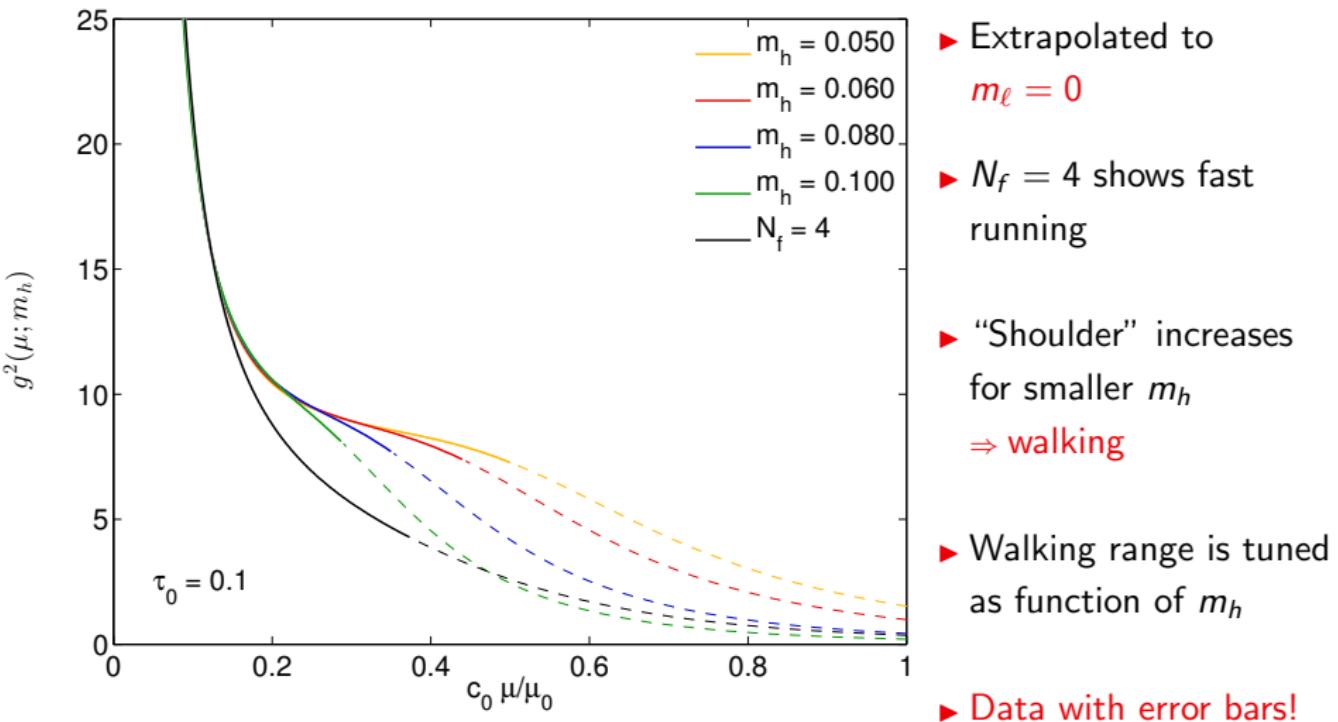
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Running coupling form gradient flow: 4+8 flavors



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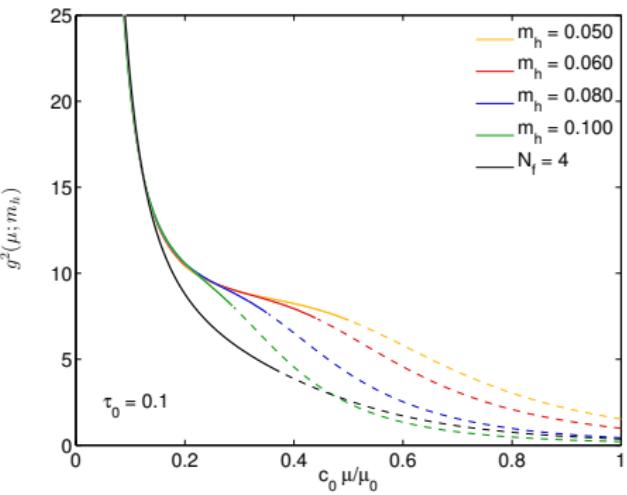
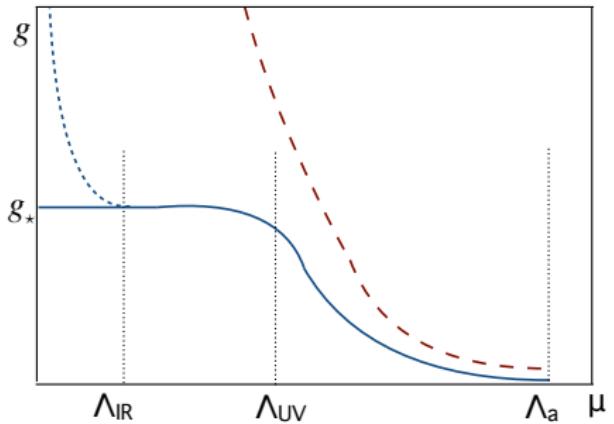
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Expectation vs. numerical data (at $m_\ell = 0$)



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Conformal systems: Wilson RG predicts hyperscaling

- ▶ Scale change: $\mu \rightarrow \mu' = \mu/b$, with $b > 1$
- ▶ Transforms the bare masses $\hat{m} = am$:

$$\hat{m}(\mu) \rightarrow \hat{m}(\mu') = b^{y_m} \hat{m}(\mu) \text{ (increase)}$$

- ▶ The bare coupling approaches its fixed point: $g \rightarrow g^*$
- ▶ Any 2-point correlator:

$$C_H(t; g, \hat{m}, \mu) \rightarrow b^{-2y_H} C_H(t/b; g_i^*, b^{y_m} \hat{m}, \mu)$$

- ▶ Now $C_H(t) \propto \exp(-M_H t)$
 $\Rightarrow aM_H \propto (\hat{m})^{1/y_m}$ (hyperscaling)
- ▶ Likewise amplitudes (F_π) show hyperscaling $\Rightarrow M_H/F_\pi$ are constant

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Hyperscaling in mass-split systems

- ▶ Light flavors of mass \hat{m}_ℓ and heavy flavors of mass \hat{m}_h :

$$\begin{aligned} C_H(t; g, \hat{m}_h, \hat{m}_\ell, \mu) &\rightarrow b^{-2y_H} C_H(t/b; g^*, b^{y_m} \hat{m}_h, b^{-y_m} \hat{m}_\ell, \mu) \\ &\equiv b^{-2y_H} C_H(t/b; g^*, b^{y_m} \hat{m}_h, \hat{m}_\ell / \hat{m}_h, \mu) \end{aligned}$$

Since $C_H(t) \propto \exp(-M_H t)$

$$a M_H \propto (\hat{m})^{1/y_m} f_H(m_\ell / m_h)$$

with $f_H(m_\ell / m_h)$ a universal function

- ▶ M_H can be build up of only light, light and heavy, or only heavy flavors
- ▶ Does not follow a system with only N_ℓ (large m_h)
or N_f degenerate (conformal) flavors

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Hyperscaling in mass-split systems

- Masses / amplitudes scale as

$$aM_H \propto (\hat{m})^{1/y_m} f_H(m_\ell/m_h)$$

- Ratios are universal functions of m_ℓ/m_h

$$M_{H_1}/M_{H_2} = \Phi_H(m_\ell/m_h)$$

$$M_H/F_\pi = \tilde{\Phi}_H(m_\ell/m_h)$$

- Chiral limit ($m_\ell = 0$): dimensionless ratios are independent of m_h
- Known F_π : rest of the spectrum is predicted – no free parameters
- ~~ There however corrections to scaling

⇒ Very different from QCD!

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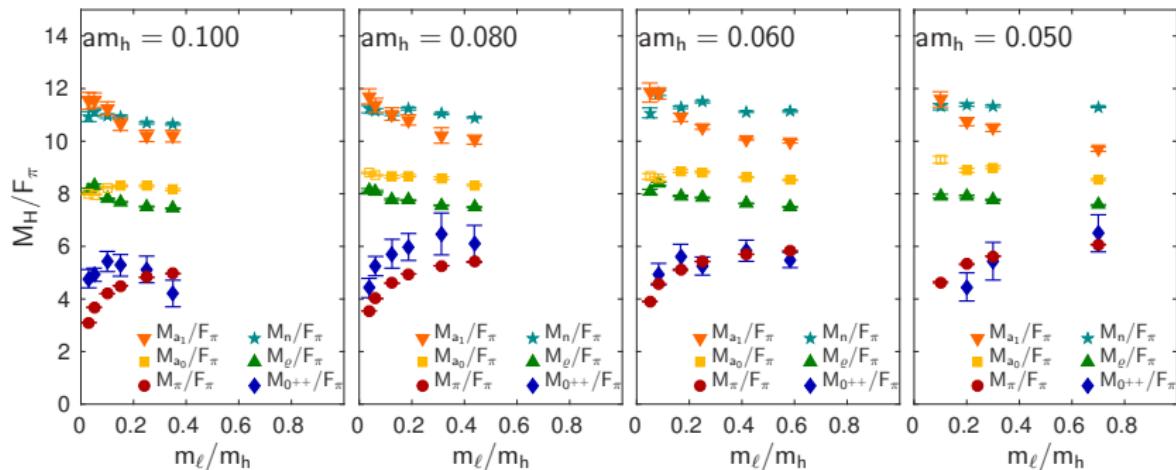
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Light-light spectrum: ratios of M_H/F_π



- ▶ Pion, rho, a_0 , a_1 , nucleon, and 0^{++} scalar
- ▶ 0^{++} is light ($M_{0^{++}} < M_\rho$), it tracks the pion. Chiral limit?
- ▶ M_π/F_π bends down \Rightarrow indicates system is chirally broken

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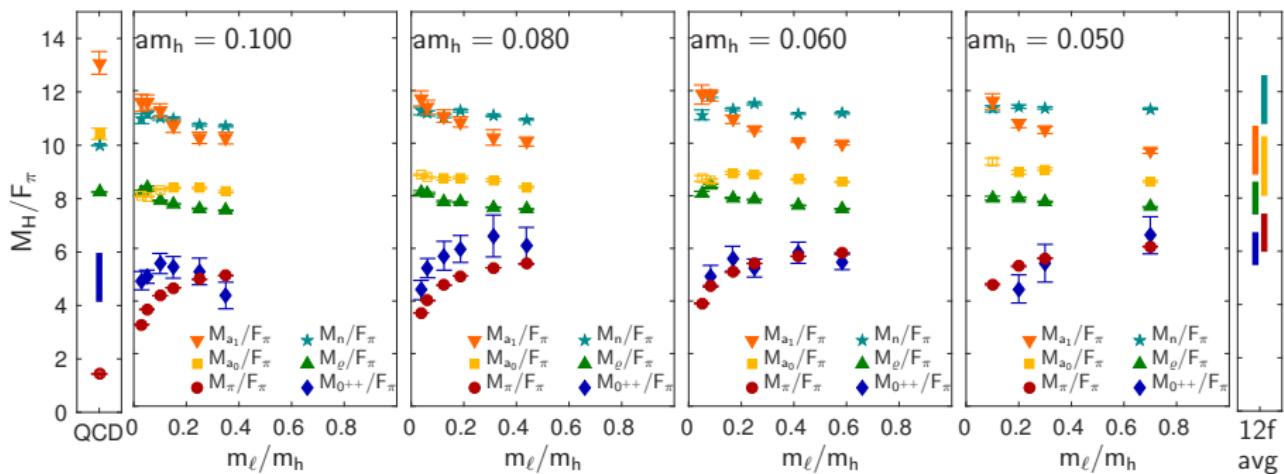
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Light-light spectrum: ratios of M_H/F_π



- ▶ For growing m_h and $m_\ell \rightarrow 0$ we approach QCD [PDG values]
- ▶ For decreasing m_h and $m_\ell \sim m_h$ we approach degenerate 12 flavors
 - [LatHC PLB 703(2011) 348] [LatKMI PRD86 (2012) 059903][LatKMI PRL 111 (2013) 162001]
 - [Cheng et al. PRD 90 (2014) 014509]

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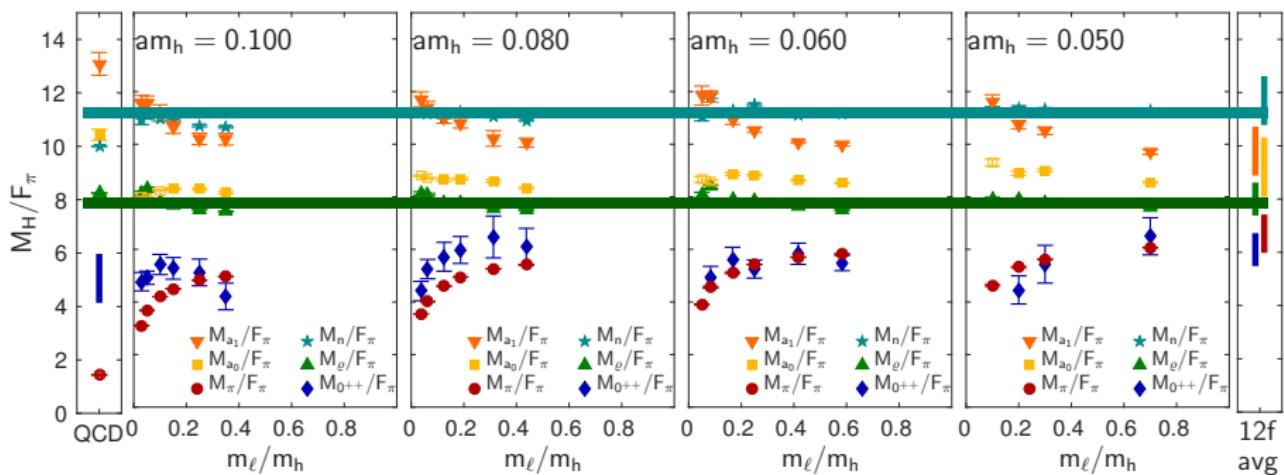
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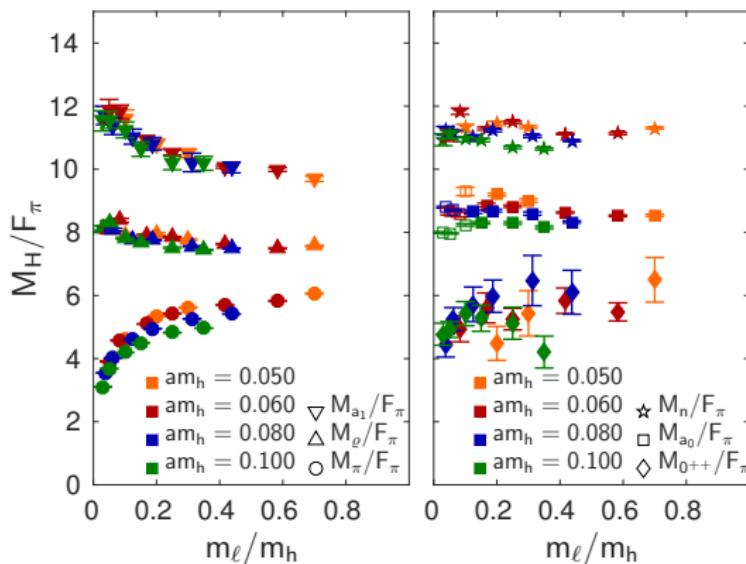
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Light-light spectrum: ratios of M_H/F_π



- ▶ Ratios appear largely independent of m_h !
- ▶ M_ρ/F_π and M_n/F_π also show very little dependence on m_ℓ

Hyperscaling at work



- ▶ $M_n/F_\pi \approx 11$
 - ▶ $M_\varrho/F_\pi \approx 8$
 - ▶ $M_{0^{++}}/F_\pi \approx 4 - 5$
 (taking the chiral limit
 is difficult but 0^{++} well
 separated from the ϱ)

experimental findings
○○○○○

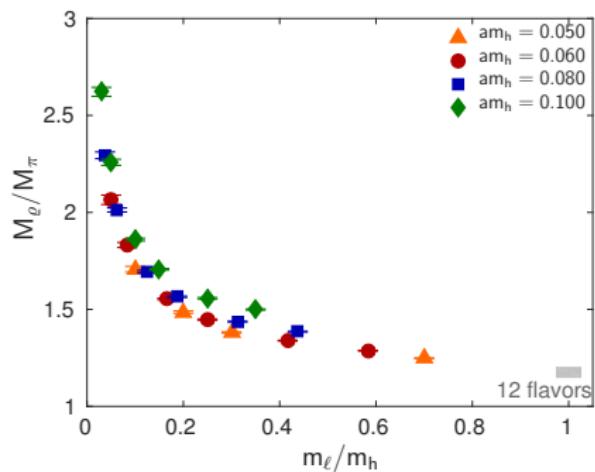
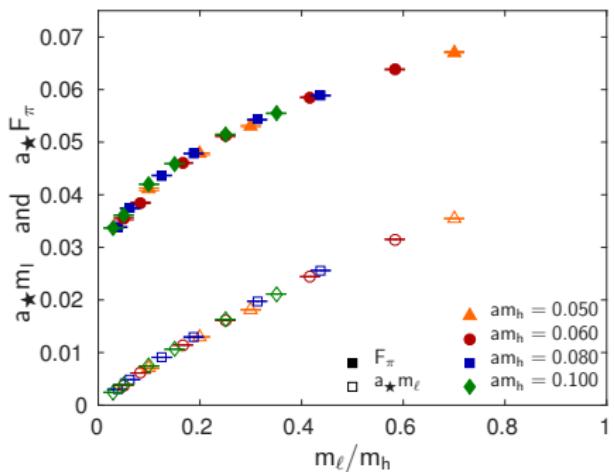
composite Higgs models
○○○○○○○○○○

running coupling
○○○○

spectrum
○○○○○●○○○

summary
○○

The system is chirally broken



► $a_\star F_\pi$ is finite

► 4-flavors (QCD-like): ratio diverges
► 12-flavors: almost constant ratio
[Cheng et al. 2014]

experimental findings
○○○○○

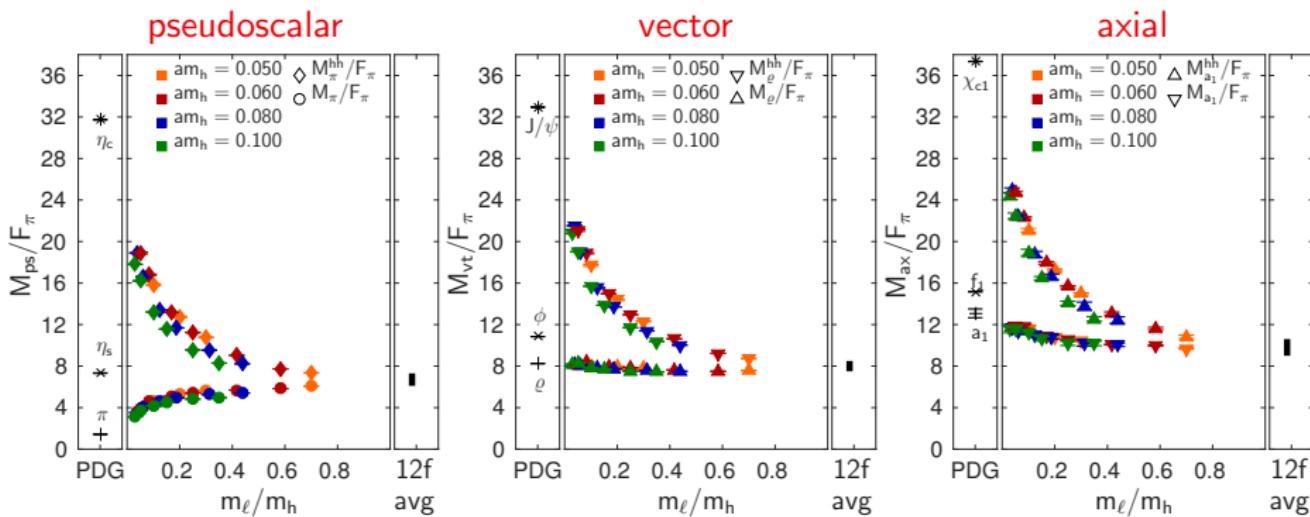
composite Higgs models
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running coupling
○○○○

spectrum
○○○○○●○○

summary
○○

Light-light and heavy-heavy spectrum



- ▶ 4+8 heavy-heavy spectrum is not QCD-like; QCD is not hyperscaling
- ▶ M^{hh}/F_π increases but F_π is finite in the chiral limit
- ▶ $M_\rho^{hh} \sim 3M_\rho \Rightarrow$ could be accessible

experimental findings
○○○○○

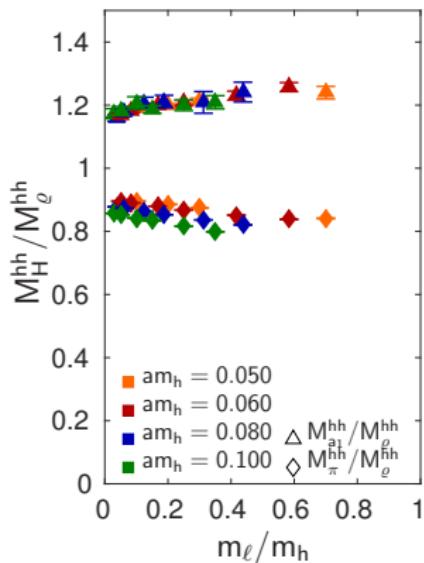
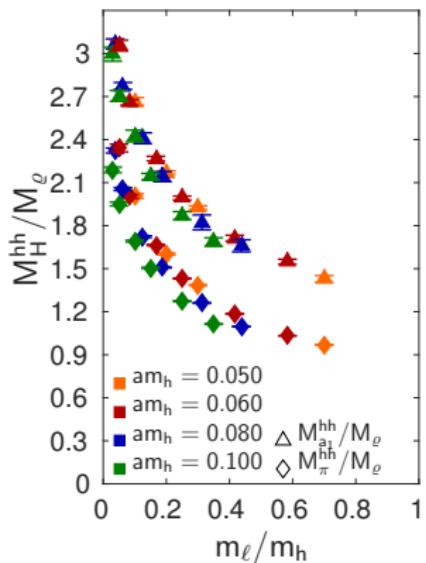
composite Higgs models
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running coupling
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spectrum
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summary
○○

Other choices for forming ratios



- ▶ Heavy-heavy states increase mostly due to light-light quantity in denominator

experimental findings
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composite Higgs models
○○○○○○○○○○

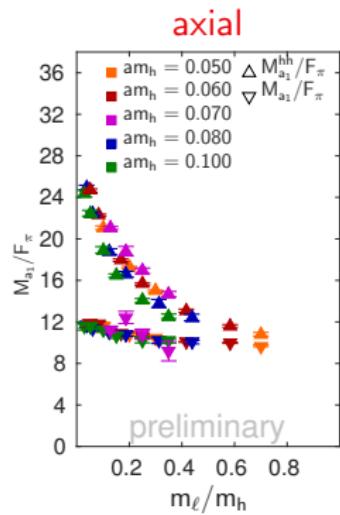
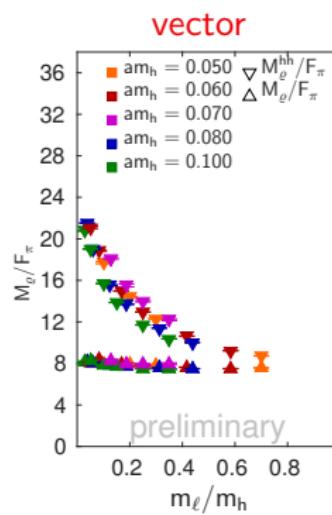
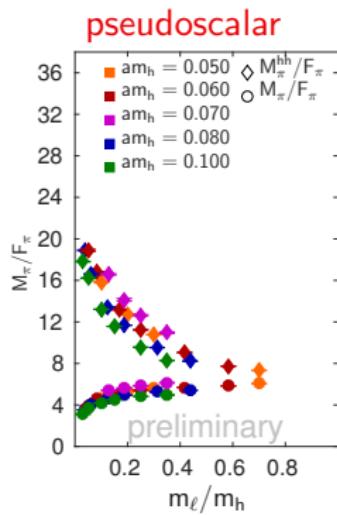
running coupling
○○○○

spectrum
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summary
○○

Irrelevance of coupling g

- ▶ Data with $am_h = 0.100, 0.080, 0.060, 0.050$ simulated at $\beta = 4.0$
- ▶ Additional simulations at $\beta = 4.4$ and $am_h = 0.070$



experimental findings
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composite Higgs models
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running coupling
○○○○

spectrum
○○○○○○○○○○

summary
○○

summary

Concluding remarks

- ▶ Mass-split models are great to explore near conformal dynamics
- ▶ Allow to investigate pNGB and dilaton-like Higgs scenarios
- ▶ Limiting cases of 4 and 12 flavors help to understand what is happening
- ▶ 4+8 is chosen for convenience of unrooted staggered fermions
 - $N_f = 12$ is above the conformal window
 - its anomalous dimension is however small $\gamma_m \approx 0.25$
- ▶ Walking gauge coupling (controlled by m_h)
- ▶ Hyperscaling ⇒ predictive
- ▶ Heavy-heavy (and heavy-light) spectrum accessible but **not** QCD-like
- ▶ **Future:** decay constants, $\pi - \pi$ scattering, etc.

experimental findings
○○○○○

composite Higgs models
○○○○○○○○○○

running coupling
○○○○

spectrum
○○○○○○○○○○

summary
○●

Resources and Acknowledgments

USQCD: Ds, Bc, and pi0 cluster (Fermilab)

BU: engaging (MGHPCC)

XSEDE: Stampede (TACC) and SuperMic (LSU)

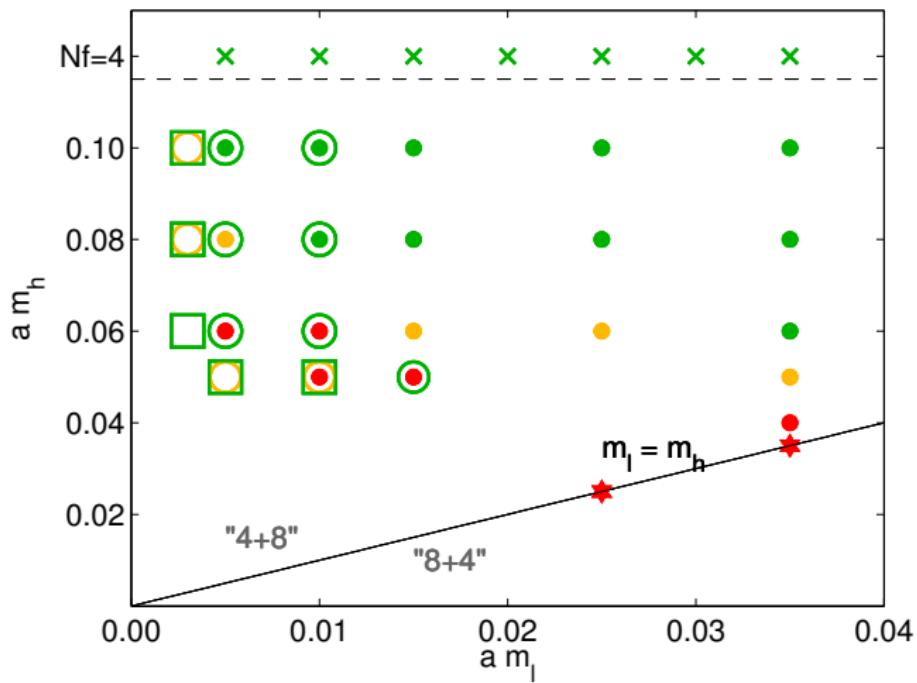


appendix

On the lattice

- ▶ Setup
 - ▶ SU(3) gauge group
 - ▶ Fundamental adjoint gauge action with $\beta_a = -\beta/4$
[Cheng et al. arXiv:1311.1287][Cheng et al. PRD 90 (2014) 014509]
 - ▶ nHYP smeared staggered Fermions [Hasenfratz et al. JHEP 05 (2007) 029]
 - ▶ Most simulations/measurements performed with FUEL [J. Osborn]
- ▶ Goals
 - ▶ Explore near conformal or conformal dynamics
 - ▶ Compute the iso-singlet 0^{++}
- ▶ References
 - [JETP 120 (2015) 3, 423] [PoS Lattice2014 254] [CCP proceedings 2014] [PRD 93 (2016) 075028]
[arXiv:1609.01401] (a longer, detailed paper is in preparation)

Performed simulations ($\beta = 4.0$)



► Symbols indicate volumes and colors
finite volume effects

red: squeezed
yellow: marginal
green: OK

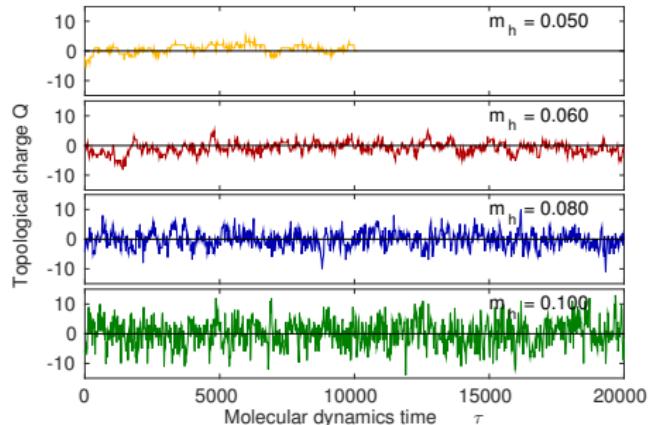
$\square: 48^3 \times 96$
or $36^3 \times 64$

$\circ: 32^3 \times 64$

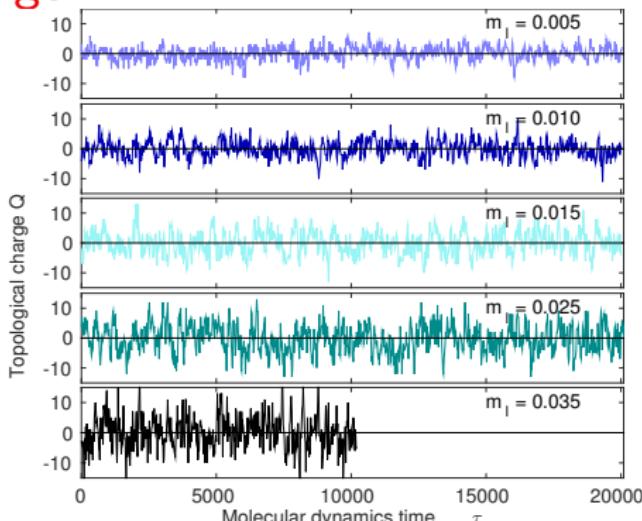
$\bullet: 24^3 \times 48$

► Up to 40k MDTU

History of the topological charge



► $am_\ell = 0.010$ fixed



► $am_h = 0.080$ fixed

- Frequency of tunneling slows down and amplitude of oscillations reduces
 - when reducing m_h for fixed m_ℓ
 - when reducing m_ℓ for fixed m_h (milder effect)

Disconnected spectrum: the 0^{++} scalar

Numerical measurement on the lattice

- ▶ 6 U(1) sources with dilution on each time slice, color and even/odd spatially
- ▶ Variance reduced $\langle \bar{\psi} \psi \rangle$

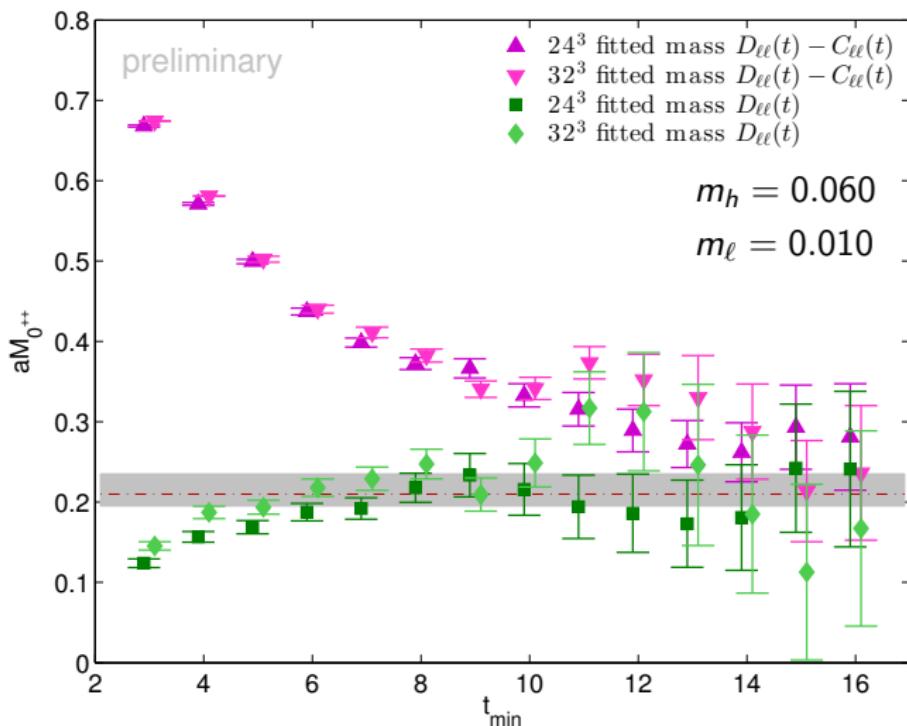
Analysis strategy

- ▶ Correlated fit to both parity states (staggered)
- ▶ **Vacuum subtraction** introduces very large uncertainties
- ▶ Advantageous to fit additional constant

$$C(t) = c_{0^{++}} \cosh \left(M_{0^{++}} \left(\frac{N_T}{2} - t \right) \right) + c_{\pi_{sc}} (-1)^t \cosh \left(M_{\pi_{sc}} \left(\frac{N_T}{2} - t \right) \right) + \nu$$

- ▶ Equivalent to fitting the finite difference: $C(t+1) - C(t)$

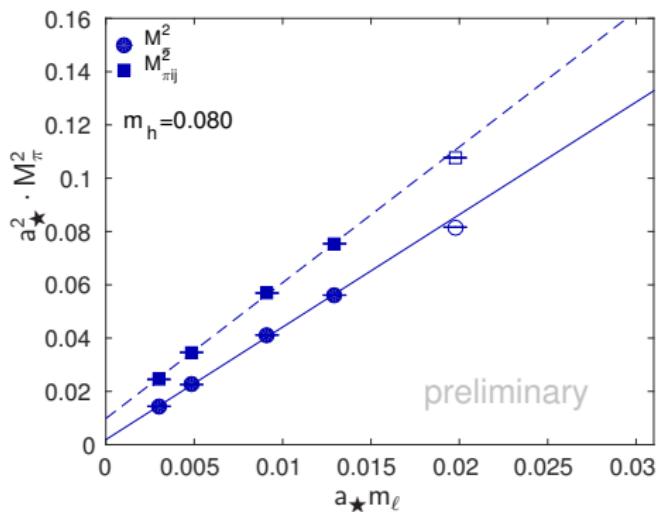
Comparison of $D_{\ell\ell}$ and $D_{\ell\ell} - C_{\ell\ell}$



- ▶ For $t \rightarrow \infty$: $D_{\ell\ell}$ and $D_{\ell\ell} - C_{\ell\ell}$ should agree (up to mixing effects)
- ▶ Compare fits with different t_{\min} and $t_{\max} = N_T/2$
- ▶ Compare results for two volumes
- ⇒ Consistent results!

Pion taste splitting

- ▶ Taste splitting is artifact of staggered fermions
- ▶ In QCD modern, smeared staggered actions show small taste splitting effects
- ▶ Taste splitting is typically constant w.r.t. m_ℓ



- ▶ Taste splitting increases for larger m_ℓ when reducing m_h

Magic 8

