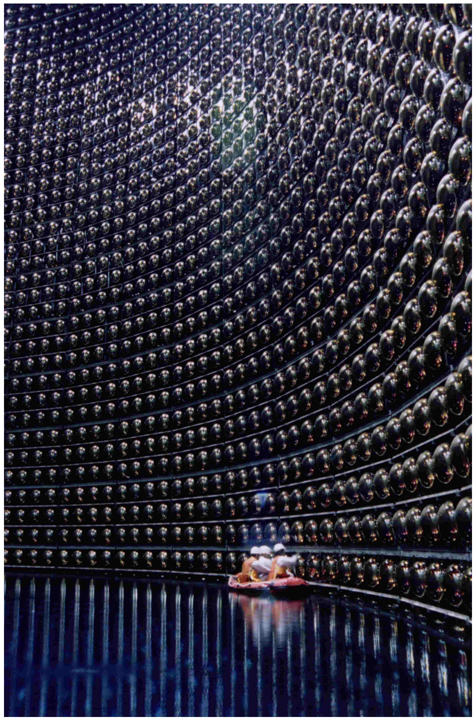
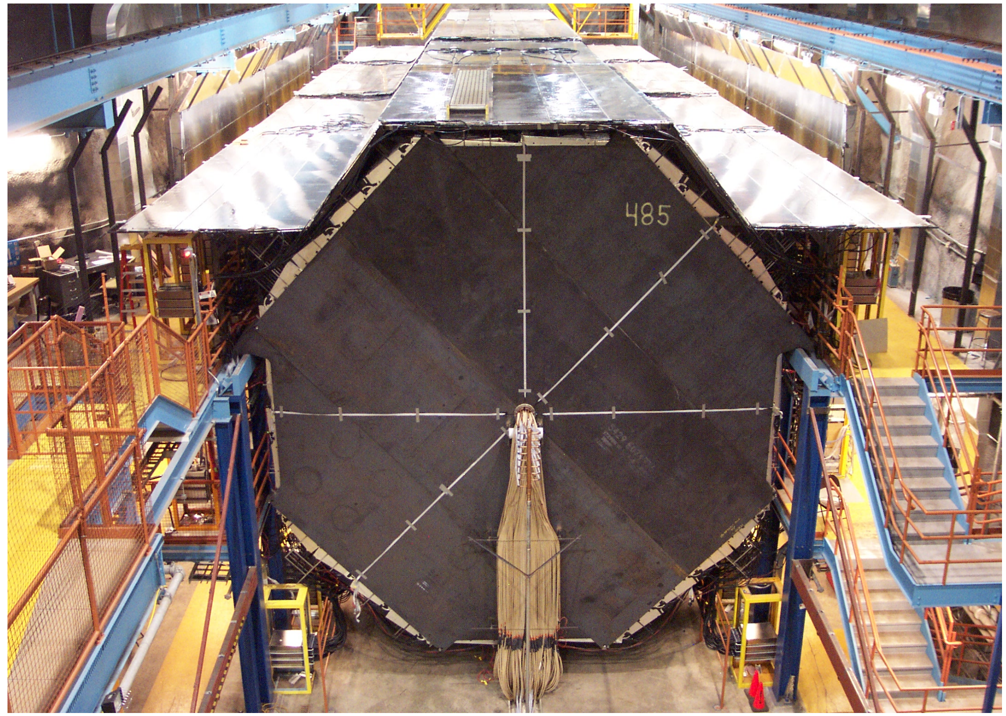
Particle Physics

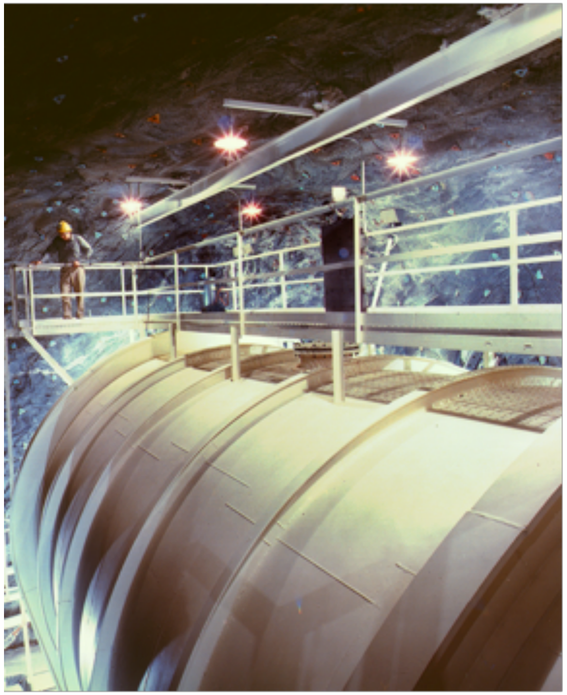
Handout from Prof. Mark Thomson's lectures
Adapted to UZH by Prof. Canelli and Prof. Serra





Handout 11 : Neutrino Oscillations

Solar neutrino problem



1970s Homestake experiment:
Solar neutrino problem

Reaction	Abbr.	Flux ($cm^{-2}\ s^{-1}$)
$pp \to d\,e^+\,\nu$	pp	$5.97(1 \pm 0.006) \times 10^{10}$
$pe^- p \to d\,\nu$	pep	$1.41(1 \pm 0.011) \times 10^{8}$
$^3\mathrm{He}\,p \to {}^4\mathrm{He}\,e^+\nu$	hep	$7.90(1 \pm 0.15) \times 10^{3}$
$^7\mathrm{Be}\,e^- \to {}^7\mathrm{Li}\,\nu + (\gamma)$	$^7\mathrm{Be}$	$5.07(1 \pm 0.06) \times 10^{9}$
$^8\mathrm{B} \to {}^8\mathrm{Be}^*\,e^+\nu$	$^8\mathrm{B}$	$5.94(1 \pm 0.11) \times 10^{6}$
$^{13}\mathrm{N} \to {}^{13}\mathrm{C}\,e^+\nu$	$^{13}\mathrm{N}$	$2.88(1 \pm 0.15) \times 10^{8}$
$^{15}\mathrm{O} \to {}^{15}\mathrm{N}\,e^+\nu$	$^{15}\mathrm{O}$	$2.15(1^{+0.17}_{-0.16}) \times 10^{8}$
$^{17}\mathrm{F} \to {}^{17}\mathrm{O}\,e^+\nu$	$^{17}\mathrm{F}$	$5.82(1^{+0.19}_{-0.17}) \times 10^{6}$

Detector based on the reaction

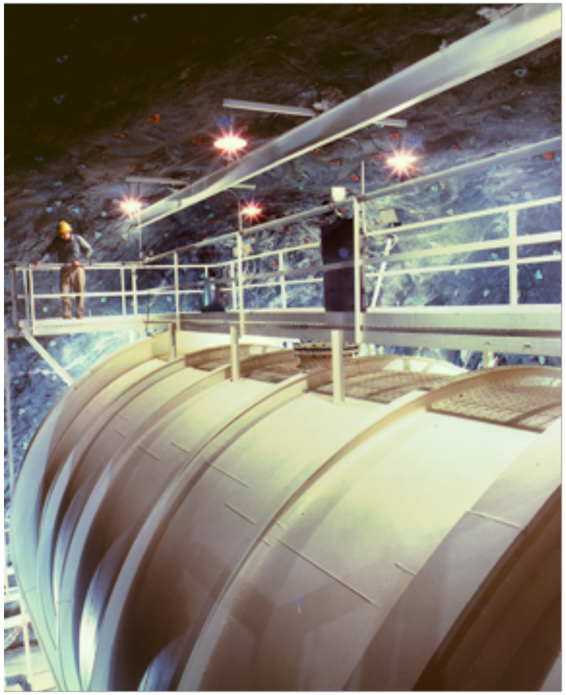
Experiment sensitive to electron neutrinos!

$$\nu_e + {}^{37}\mathrm{Cl} \mapsto {}^{37}\mathrm{Ar} + e^-$$

Measured neutrino flux

measured flux	ratio exp/BP98
$2.56 \pm 0.16 \pm 0.16$	$0.33 \pm 0.03 \pm 0.05$

Solar neutrino problem



Hypothesis:

- Neutrinos have a mass and oscillate (like neutral mesons)

- Since the experiment sensitive to electron neutrinos, the missing neutrinos have oscillated to another specie

$$\nu_e \to \nu_\mu \qquad \nu_e \to \nu_\tau$$

measured flux	ratio exp/BP98
2.56 ± 0.16 ± 0.16	0.33 ± 0.03 ± 0.05

Historical Overview

This observation was confirmed by several experiments

Experiment	measured flux	ratio exp/BP98	threshold energy	Years of running
Homestake	$2.56 \pm 0.16 \pm 0.16$	$0.33 \pm 0.03 \pm 0.05$	0.814 MeV	1970-1995
Kamiokande	$2.80 \pm 0.19 \pm 0.33$	$0.54 \pm 0.08\ ^{+0.10}_{-0.07}$	7.5 MeV	1986-1995
SAGE	$75 \pm 7 \pm 3$	$0.58 \pm 0.06 \pm 0.03$	0.233 MeV	1990-2006
Gallex	$78 \pm 6 \pm 5$	$0.60 \pm 0.06 \pm 0.04$	0.233 MeV	1991-1996
Super-Kamiokande	$2.35 \pm 0.02 \pm 0.08$	$0.465 \pm 0.005\ ^{+0.016}_{-0.015}$ (BP00)	5.5 (6.5) MeV	1996-
GNO	$66 \pm 10 \pm 3$	$0.51 \pm 0.08 \pm 0.03$	0.233 MeV	1998-
SNO	$1.68 \pm 0.06 \pm\ ^{+0.08}_{-0.09}$ (CC) $2.35 \pm 0.22 \pm 0.15$ (ES) $4.94 \pm 0.21\ ^{+0.38}_{-0.34}$ (NC)		6.75 MeV	1999-

In addition to solar neutrino problem, neutrino disappearance has been observed in atmospheric neutrinos and nuclear reactors

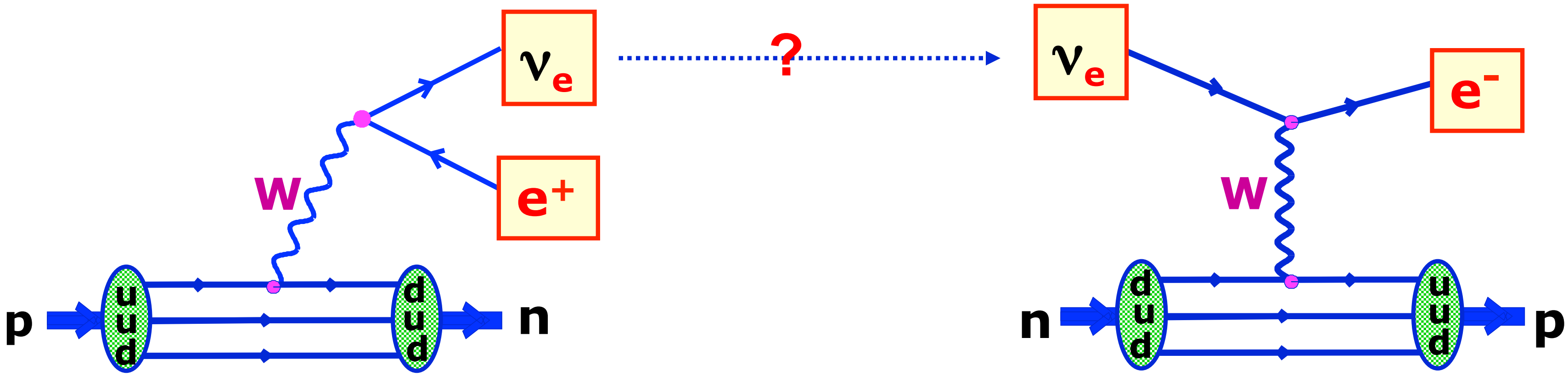
Neutrino Flavours Revisited

★ Never **directly** observe neutrinos – can only detect them by their weak interactions. Hence by **definition** ν_e is the neutrino state produced along with an electron. Similarly, charged current weak interactions of the state ν_e produce an electron

ν_e, ν_μ, ν_τ = weak eigenstates

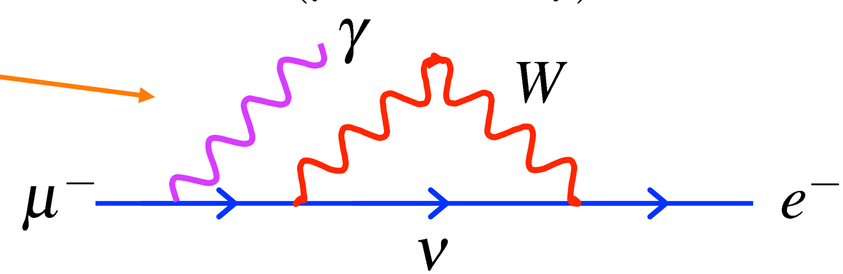
★ For many years, assumed that ν_e, ν_μ, ν_τ were massless fundamental particles

- **Experimental evidence:** neutrinos produced along with an electron always produced an electron in CC Weak interactions, etc.



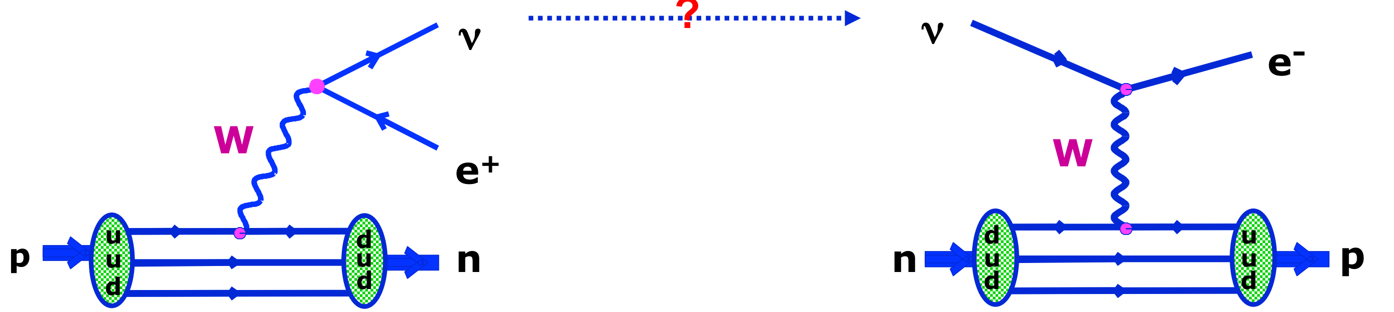
- **Experimental evidence:** absence $\mu^- \to e^- \gamma$ $\quad \mathrm{BR}(\mu^- \to e^- \gamma) < 10^{-11}$

Suggests that ν_e and ν_μ are distinct particles otherwise decay could go via:

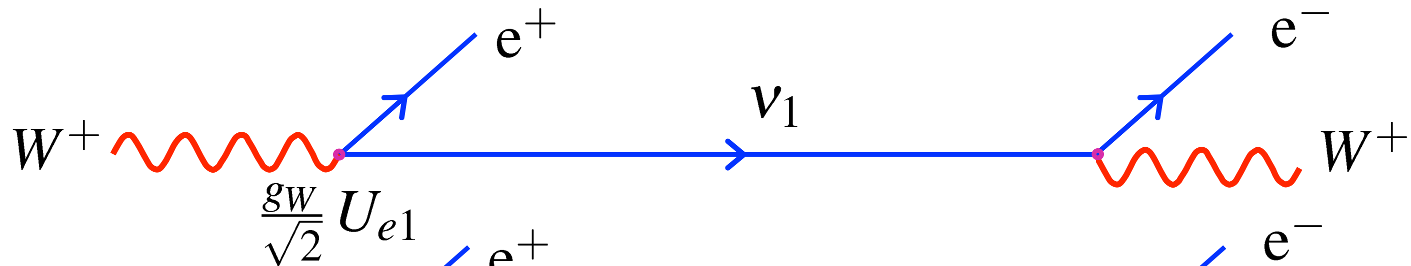


Mass Eigenstates and Weak Eigenstates

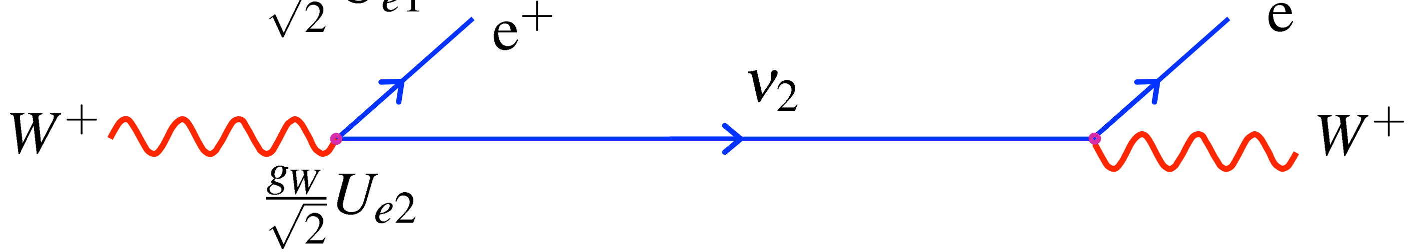
- ★ The essential feature in understanding the physics of neutrino oscillations is to understand what is meant by weak eigenstates and mass eigenstates ν_1, ν_2
- ★ Suppose the process below proceeds via two fundamental particle states



i.e.



and



- ★ Can't know which **mass eigenstate** (fundamental particle ν_1, ν_2) was involved
- ★ In Quantum mechanics treat as a coherent state $\psi = \nu_e = U_{e1}\nu_1 + U_{e2}\nu_2$
- ★ ν_e represents the wave-function of the coherent state produced along with an electron in the weak interaction, i.e. the **weak eigenstate**

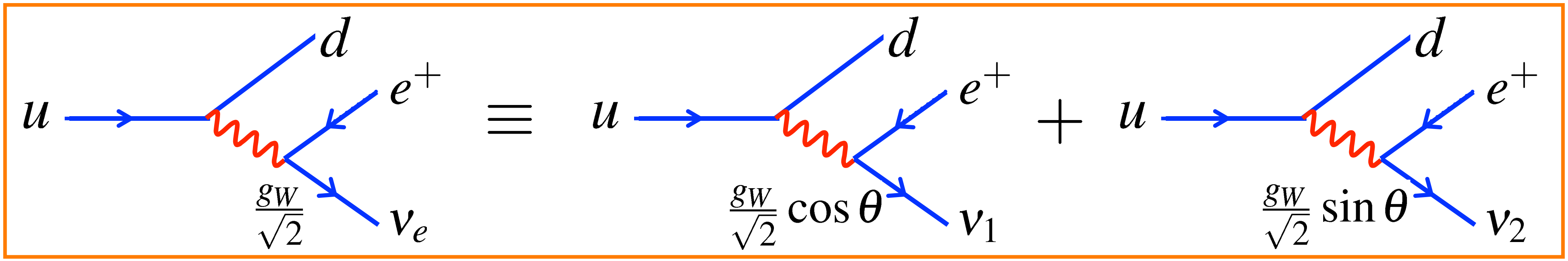
Neutrino Oscillations for Two Flavours

- ★ Neutrinos are produced and interact as weak eigenstates, $\nu_e,\ \nu_\mu$
- ★ The weak eigenstates as coherent linear combinations of the fundamental "mass eigenstates" $\nu_1,\ \nu_2$
- ★ The mass eigenstates are the free particle solutions to the wave-equation and will be taken to propagate as plane waves

$$|\nu_1(t)\rangle = |\nu_1\rangle e^{i\vec{p}_1.\vec{x}-iE_1t} \qquad |\nu_2(t)\rangle = |\nu_2\rangle e^{i\vec{p}_2.\vec{x}-iE_2t}$$

- ★ The weak and mass eigenstates are related by the unitary 2x2 matrix

$$\begin{pmatrix}\nu_e\\ \nu_\mu\end{pmatrix} = \begin{pmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{pmatrix}\begin{pmatrix}\nu_1\\ \nu_2\end{pmatrix} \qquad (1)$$



- ★ Equation (1) can be inverted to give

$$\begin{pmatrix}\nu_1\\ \nu_2\end{pmatrix} = \begin{pmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{pmatrix}\begin{pmatrix}\nu_e\\ \nu_\mu\end{pmatrix} \qquad (2)$$

- Suppose at time $t=0$ a neutrino is produced in a pure ν_e state, e.g. in a decay $u \to d e^+ \nu_e$

$$|\psi(0)\rangle = |\nu_e\rangle = \cos\theta|\nu_1\rangle + \sin\theta|\nu_2\rangle$$

- Take the z-axis to be along the neutrino direction
- The wave-function evolves according to the time-evolution of the **mass eigenstates** (free particle solutions to the wave equation)

$$|\psi(t)\rangle = \cos\theta|\nu_1\rangle e^{-ip_1.x} + \sin\theta|\nu_2\rangle e^{-ip_2.x}$$

where $p_i.x = E_i t - \vec{p}_i.\vec{x} = E_i t - |\vec{p}_i|z$

- Suppose the neutrino interacts in a detector at a distance **L** and at a time **T**

$$\phi_i = p_i.x = E_i T - |\vec{p}_i|L$$

gives

$$|\psi(L,T)\rangle = \cos\theta|\nu_1\rangle e^{-i\phi_1} + \sin\theta|\nu_2\rangle e^{-i\phi_2}$$

★ Expressing the mass eigenstates, $|\nu_1\rangle, |\nu_2\rangle$, in terms of weak eigenstates (eq 2)

$$|\psi(L,T)\rangle = \cos\theta(\cos\theta|\nu_e\rangle - \sin\theta|\nu_\mu\rangle)e^{-i\phi_1} + \sin\theta(\sin\theta|\nu_e\rangle + \cos\theta|\nu_\mu\rangle)e^{-i\phi_2}$$

$$|\psi(L,T)\rangle = |\nu_e\rangle(\cos^2\theta e^{-i\phi_1} + \sin^2\theta e^{-i\phi_2}) + |\nu_\mu\rangle\sin\theta\cos\theta(-e^{-i\phi_1} + e^{-i\phi_2})$$

★ If the masses of $|\nu_1\rangle, |\nu_2\rangle$ **are the same**, the mass eigenstates remain in phase, $\phi_1 = \phi_2$, and the state remains the linear combination corresponding to $|\nu_e\rangle$ and in a weak interaction will produce an electron

★ If the masses are different, the wave-function no longer remains a pure $|\nu_e\rangle$

$$
\begin{aligned}
P(\nu_e \to \nu_\mu) &= |\langle \nu_\mu | \psi(L,T)\rangle|^2 \\
&= \cos^2\theta \sin^2\theta(-e^{-i\phi_1} + e^{-i\phi_2})(-e^{+i\phi_1} + e^{+i\phi_2}) \\
&= \frac{1}{4}\sin^2 2\theta(2 - 2\cos(\phi_1 - \phi_2)) \\
&= \sin^2 2\theta \sin^2\left(\frac{\phi_1 - \phi_2}{2}\right)
\end{aligned}
$$

★ The treatment of the phase difference

$$\Delta\phi_{12} = \phi_1 - \phi_2 = (E_1 - E_2)T - (|p_1| - |p_2|)L$$

in most text books is dubious. Here we will be more careful…

★ One could assume $|p_1| = |p_2| = \mathrm{p}$ in which case

$$\Delta\phi_{12} = (E_1 - E_2)T = \left[(\mathrm{p}^2 + m_1^2)^{1/2} - (\mathrm{p}^2 + m_2^2)^{1/2}\right] L \qquad L \approx (c)T$$

$$\Delta\phi_{12} = \mathrm{p}\left[\left(1+\frac{m_1^2}{\mathrm{p}^2}\right)^{1/2} - \left(1+\frac{m_2^2}{\mathrm{p}^2}\right)^{1/2}\right] L \approx \frac{m_1^2 - m_2^2}{2\mathrm{p}} L$$

★ **However we have neglected that fact that for the same momentum, different mass eigenstates will propagate at different velocities and be observed at different times**

★ **The full derivation requires a wave-packet treatment and gives the same result**

★ **Nevertheless it is worth noting that the phase difference can be written**

$$\Delta\phi_{12} = (E_1 - E_2)T - \left(\frac{|p_1|^2 - |p_2|^2}{|p_1| + |p_2|}\right) L$$

$$\Delta\phi_{12} = (E_1 - E_2)\left[T - \left(\frac{E_1 + E_2}{|p_1| + |p_2|}\right) L\right] + \left(\frac{m_1^2 - m_2^2}{|p_1| + |p_2|}\right) L$$

★ **The first term on the RHS vanishes if we assume** $E_1 = E_2$ **or** $\beta_1 = \beta_2$

in all cases

$$\boxed{\Delta\phi_{12} = \frac{m_1^2 - m_2^2}{2\mathrm{p}} L = \frac{\Delta m^2}{2E} \mathbf{L}}$$

★ **Hence the two-flavour oscillation probability is:**

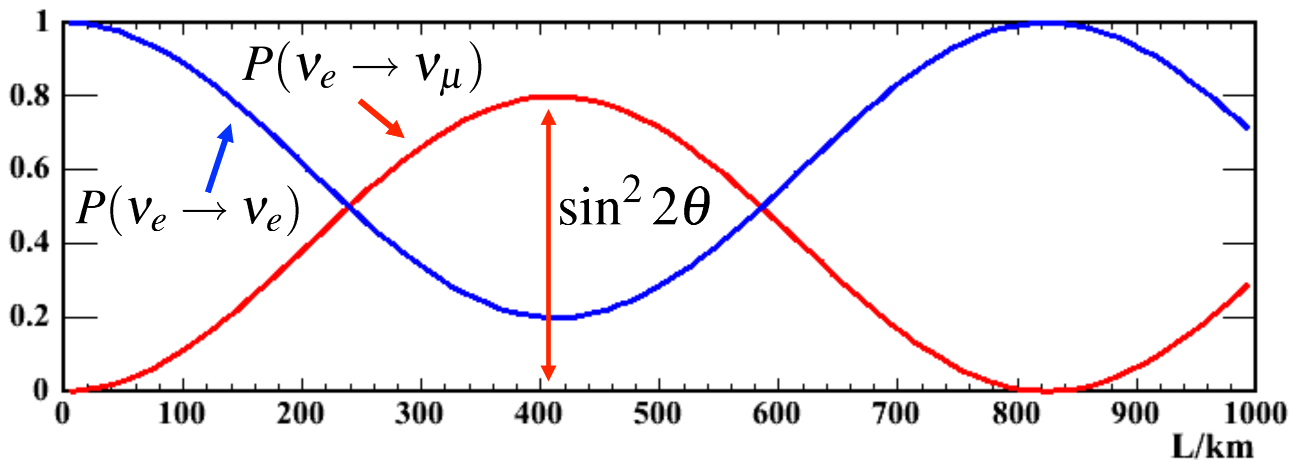
$$P(\nu_e \to \nu_\mu) = \sin^2 2\theta \sin^2\left(\frac{\Delta m^2_{21} L}{4E}\right)$$

with $$\Delta m^2_{21} = m_2^2 - m_1^2$$

★ **The corresponding two-flavour survival probability is:**

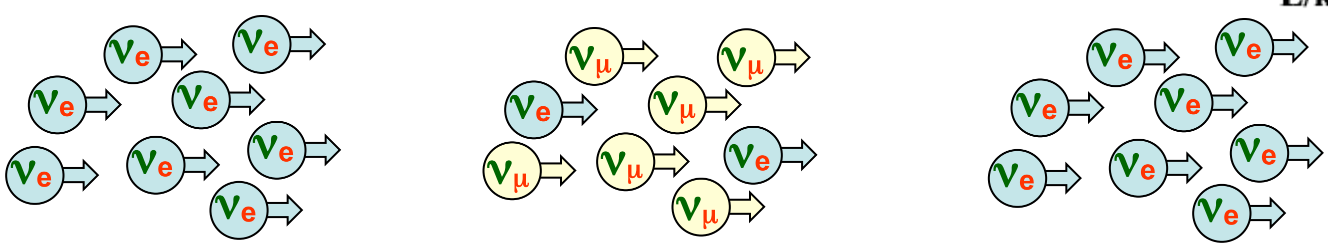
$$P(\nu_e \to \nu_e) = 1 - \sin^2 2\theta \sin^2\left(\frac{\Delta m^2_{21} L}{4E}\right)$$

• **e.g.** $\Delta m^2 = 0.003\,\mathrm{eV}^2, \qquad \sin^2 2\theta = 0.8, \qquad E_\nu = 1\,\mathrm{GeV}$



• **wavelength**

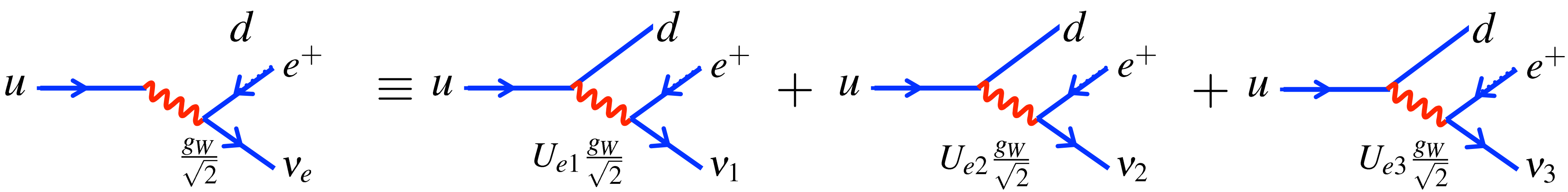
$$\lambda_{osc} = \frac{4\pi E}{\Delta m^2}$$



Neutrino Oscillations for Three Flavours

★ It is simple to extend this treatment to three generations of neutrinos.
★ In this case we have:

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$



★ The 3x3 Unitary matrix U is known as the Pontecorvo-Maki-Nakagawa-Sakata matrix, usually abbreviated **PMNS**

★ Note : has to be unitary to conserve probability

•Using $U^\dagger U = I \quad \Rightarrow \quad U^{-1} = U^\dagger = (U^*)^T$

gives

$$\begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} = \begin{pmatrix} U^*_{e1} & U^*_{\mu 1} & U^*_{\tau 1} \\ U^*_{e2} & U^*_{\mu 2} & U^*_{\tau 2} \\ U^*_{e3} & U^*_{\mu 3} & U^*_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix}$$

Neutrino oscillation (3 flavours)

- Before we had

$$P(\nu_e \to \nu_e) = 1 - \sin^2 2\theta \sin^2\left(\frac{\Delta m_{21}^2 L}{4E}\right)$$

with $\Delta m_{21}^2 = m_2^2 - m_1^2$

$$P(\nu_e \to \nu_\mu) = \sin^2 2\theta \sin^2\left(\frac{\Delta m_{21}^2 L}{4E}\right)$$

$$\begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix}$$

- This can be written as

$$P(\nu_e \to \nu_e) = 1 - 4|U_{e1}|^2|U_{e2}|^2 \sin^2\Delta_{21}$$

$$\Delta_{21} = \frac{(m_2^2 - m_1^2)L}{4E} = \frac{\Delta m_{21}^2 L}{4E}$$

- This can be generalized to the 3 flavours as

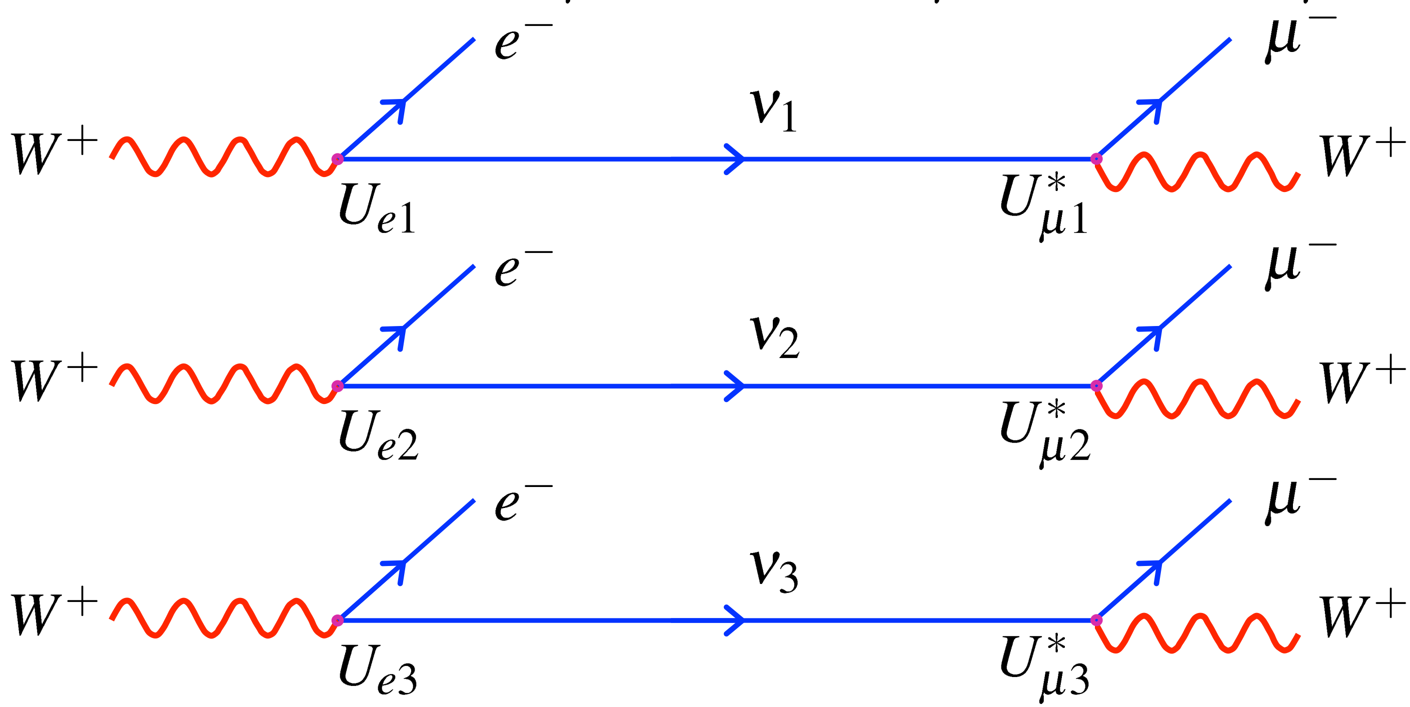
$$P(\nu_e \to \nu_e) = 1 - 4|U_{e1}|^2|U_{e2}|^2 \sin^2\Delta_{21} - 4|U_{e1}|^2|U_{e3}|^2 \sin^2\Delta_{31} - 4|U_{e2}|^2|U_{e3}|^2 \sin^2\Delta_{32}$$

★ Note that since we only have three neutrino generations there are only two independent mass-squared differences, i.e.

$$m_3^2 - m_1^2 = (m_3^2 - m_2^2) + (m_2^2 - m_1^2)$$

• The representation of neutrino oscillation is the following

$$P(\nu_e \to \nu_\mu) = |\langle \nu_\mu | \psi(L) \rangle|^2$$
$$= |U_{e1}U^*_{\mu 1}e^{-i\phi_1} + U_{e2}U^*_{\mu 2}e^{-i\phi_2} + U_{e3}U^*_{\mu 3}e^{-i\phi_3}|^2$$



★ As before the oscillation depend on the difference in mass of the eigenstates

★ All expressions are in Natural Units, conversion to more useful units here gives:

$$\Delta_{21} = 1.27 \frac{\Delta m^2_{21}(\mathrm{eV}^2) L(\mathrm{km})}{E(\mathrm{GeV})}$$ and $$\lambda_{\mathrm{osc}}(\mathrm{km}) = 2.47 \frac{E(\mathrm{GeV})}{\Delta m^2(\mathrm{eV}^2)}$$

★The Unitarity of the PMNS matrix gives several useful relations: $UU^{\dagger} = I$

The number of free parameters are three real angles and a complex phase

- It can be shown that the oscillation probability for $\nu_e \rightarrow \nu_\mu$ is

$$\begin{aligned} P(\nu_e \rightarrow \nu_\mu) &= 2\Re\{U_{e1}U^*_{\mu 1}U^*_{e2}U_{\mu 2}[e^{-i(\phi_1-\phi_2)}-1]\} \\ &+ 2\Re\{U_{e1}U^*_{\mu 1}U^*_{e3}U_{\mu 3}[e^{-i(\phi_1-\phi_3)}-1]\} \\ &+ 2\Re\{U_{e2}U^*_{\mu 2}U^*_{e3}U_{\mu 3}[e^{-i(\phi_2-\phi_3)}-1]\} \end{aligned}$$

- The oscillation probability for $\nu_\mu \rightarrow \nu_e$ can be obtained in the same manner or by simply exchanging the labels $(e) \leftrightarrow (\mu)$

$$\begin{aligned} P(\nu_\mu \rightarrow \nu_e) &= 2\Re\{U_{\mu 1}U^*_{e1}U^*_{\mu 2}U_{e2}[e^{-i(\phi_1-\phi_2)}-1]\} \\ &+ 2\Re\{U_{\mu 1}U^*_{e1}U^*_{\mu 3}U_{e3}[e^{-i(\phi_1-\phi_3)}-1]\} \\ &+ 2\Re\{U_{\mu 2}U^*_{e2}U^*_{\mu 3}U_{e3}[e^{-i(\phi_2-\phi_3)}-1]\} \end{aligned}$$

★ Unless the elements of the PMNS matrix are real

$$P(\nu_e \rightarrow \nu_\mu) \neq P(\nu_\mu \rightarrow \nu_e)$$

- If any of the elements of the PMNS matrix are complex, neutrino oscillations are not invariant under time reversal $t \rightarrow -t$

- Consider the effects of T, CP and CPT on neutrino oscillations

T	$\nu_e \to \nu_\mu$	$\xrightarrow{\hat{T}}$	$\nu_\mu \to \nu_e$
CP	$\nu_e \to \nu_\mu$	$\xrightarrow{\hat{C}\hat{P}}$	$\overline{\nu}_e \to \overline{\nu}_\mu$
CPT	$\nu_e \to \nu_\mu$	$\xrightarrow{\hat{C}\hat{P}\hat{T}}$	$\overline{\nu}_\mu \to \overline{\nu}_e$

Note C alone is not sufficient as it transforms LH neutrinos into LH anti-neutrinos (not involved in Weak Interaction)

- If the weak interactions is invariant under CPT

$$P(\nu_e \to \nu_\mu) = P(\overline{\nu}_\mu \to \overline{\nu}_e)$$

and similarly

$$P(\nu_\mu \to \nu_e) = P(\overline{\nu}_e \to \overline{\nu}_\mu) \qquad (10)$$

- If the PMNS matrix is not purely real, then (9)

$$P(\nu_e \to \nu_\mu) \neq P(\nu_\mu \to \nu_e)$$

and from (10)

$$P(\nu_e \to \nu_\mu) \neq P(\overline{\nu}_e \to \overline{\nu}_\mu)$$

★**Hence unless the PMNS matrix is real, CP is violated in neutrino oscillations!**

Neutrino Mass Hierarchy

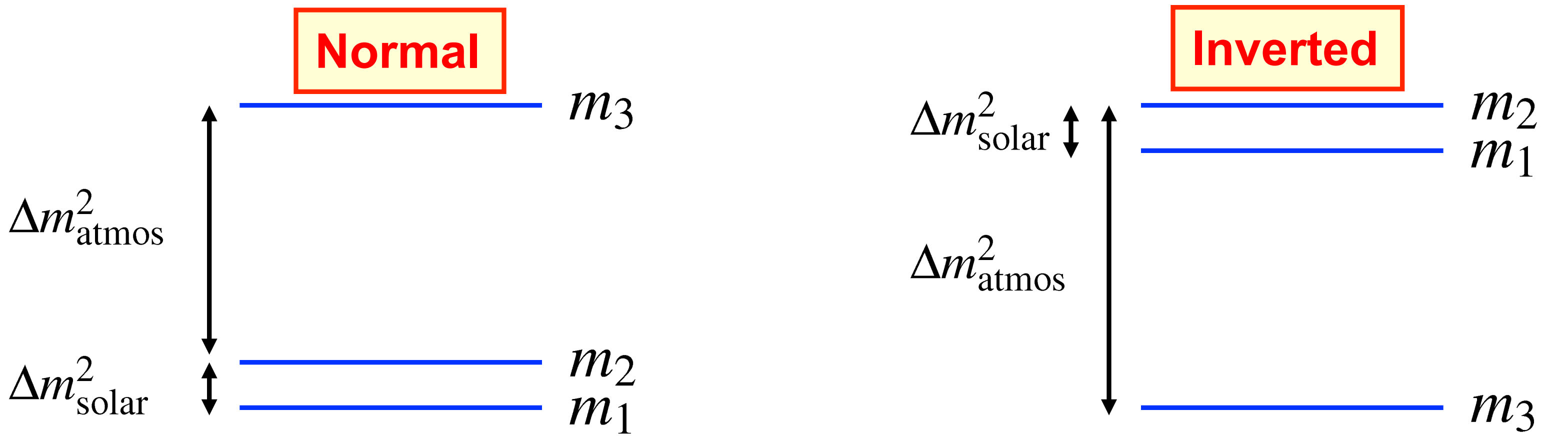
★ To date, results on neutrino oscillations only determine

$$|\Delta m^2_{ji}| = |m^2_j - m^2_i|$$

★ Two distinct and very different mass scales:

- Atmospheric neutrino oscillations : $|\Delta m^2|_{\rm atmos} \sim 2.5 \times 10^{-3}\,{\rm eV}^2$
- Solar neutrino oscillations: $|\Delta m^2|_{\rm solar} \sim 8 \times 10^{-5}\,{\rm eV}^2$

•Two possible assignments of mass hierarchy:



•In both cases: $\Delta m^2_{21} \sim 8 \times 10^{-5}\,{\rm eV}^2$ (solar)

$$|\Delta m^2_{31}| \approx |\Delta m^2_{32}| \sim 2.5 \times 10^{-3}\,{\rm eV}^2 \quad \text{(atmospheric)}$$

•Hence we can approximate $\Delta m^2_{31} \approx \Delta m^2_{32}$

Summary of Current Knowledge

★ **As for the quark matrix, the PMNS matrix has in principle one imaginary phase (not yet measured)**

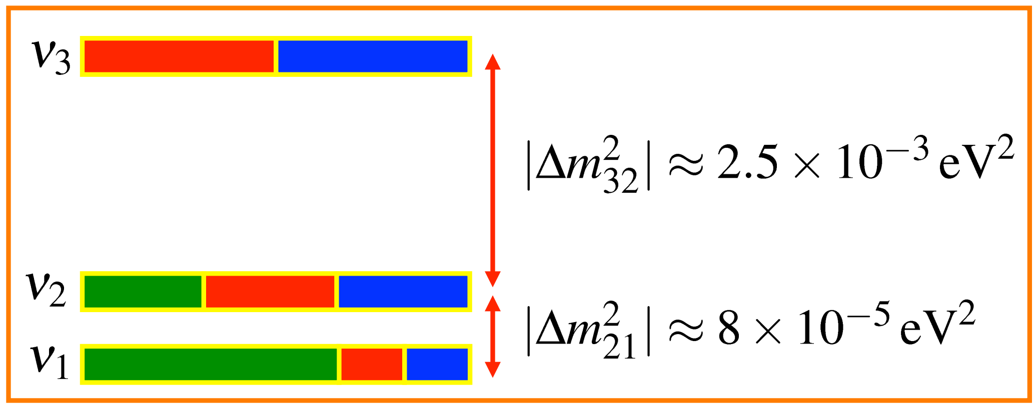
$$\begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \approx \begin{pmatrix} 0.85 & 0.53 & 0.1e^{i\delta}? \\ -0.37 & 0.60 & 0.71 \\ 0.37 & -0.60 & 0.71 \end{pmatrix}$$

★ **Have approximate expressions for mass eigenstates in terms of weak eigenstates:**

$$|\nu_3\rangle \approx \frac{1}{\sqrt{2}}(|\nu_\mu\rangle + |\nu_\tau\rangle)$$

$$|\nu_2\rangle \approx 0.53|\nu_e\rangle + 0.60(|\nu_\mu\rangle - |\nu_\tau\rangle)$$

$$|\nu_1\rangle \approx 0.85|\nu_e\rangle - 0.37(|\nu_\mu\rangle - |\nu_\tau\rangle)$$



Final Words: Neutrino Masses

- Neutrino oscillations require non-zero neutrino masses (for at least 2 neutrinos)
- But only determine **mass-squared differences** – not the masses themselves
- No direct measure of neutrino mass – only mass limits:

$$m_\nu(e) < 2\,\text{eV}; \quad m_\nu(\mu) < 0.17\,\text{MeV}; \quad m_\nu(\tau) < 18.2\,\text{MeV}$$

Note the e, μ, τ refer to charged lepton flavour in the experiment, e.g. $m_\nu(e) < 2\,\text{eV}$ refers to the limit from tritium beta-decay

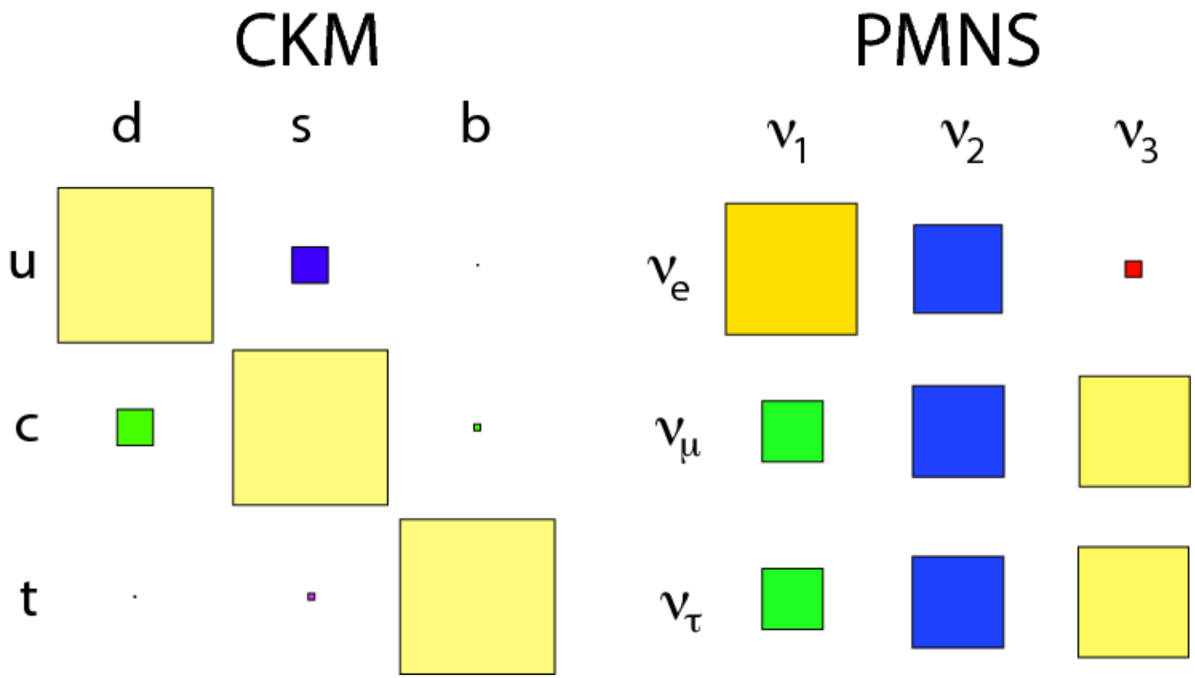
- Also from cosmological evolution infer that the sum

$$\sum_i m_{\nu_i} < \text{few eV}$$

★ 10 years ago – assumed massless neutrinos + hints that neutrinos might oscillate !

★ Now, know a great deal about massive neutrinos

Comparison CKM and PMNS matrixes



★ **The elements of these two matrixes are free parameters in the SM**
★ **The CKM is almost diagonal, while the PMNS is completely different, we do not know why**