Exercise 1. ***Dimensional regularisation***

Prove the following identities:

$$\begin{aligned} \int \frac{\mathrm{d}^d p_E}{(2\pi)^d} \frac{1}{(p_E^2+\Delta)^n} &= \frac{1}{(4\pi)^{d/2}} \frac{\Gamma\left(n-\frac{d}{2}\right)}{\Gamma(n)} \Delta^{\frac{d}{2}-n}\,, \\ \int \frac{\mathrm{d}^d p_E}{(2\pi)^d} \frac{p_E^2}{(p_E^2+\Delta)^n} &= \frac{1}{(4\pi)^{d/2}} \frac{d}{2} \frac{\Gamma\left(n-\frac{d}{2}-1\right)}{\Gamma(n)} \Delta^{1+\frac{d}{2}-n}\,. \end{aligned} \tag{1}$$

Exercise 2. ***Feynman parametrisation***

Prove by induction the formula for the Feynman parametrisation of n propagators:

$$\begin{aligned} \frac{1}{D_1^{a_1}\dots D_n^{a_n}} =& \frac{\Gamma(a_1+a_2+\ldots+a_n)}{\Gamma(a_1)\Gamma(a_2)\ldots\Gamma(a_n)} \\ &\times \int_0^1 \mathrm{d}x_1 \ldots \int_0^1 \mathrm{d}x_n \frac{\delta(1-x_1-x_2-\ldots-x_n)\; x_1^{a_1-1}x_2^{a_2-1}\ldots x_n^{a_n-1}}{[x_1D_1+\ldots+x_nD_n]^{a_1+\ldots+a_n}}\,. \end{aligned} \tag{2}$$

Exercise 3. ***Generalisation of γ_5 in d dimensions***

In four dimensions, γ_5 can be defined by its two properties:

$$\begin{aligned} &(a) \qquad \mathrm{Tr}\left(\gamma_\mu\gamma_\nu\gamma_\rho\gamma_\sigma\gamma_5\right) = \epsilon_{\mu\nu\rho\sigma}\,\mathrm{Tr}\,\mathbf{1}\,, \\ &(b) \qquad \{\gamma_\mu, \gamma_5\} = 0\,. \end{aligned} \tag{3}$$

Show that those two properties cannot be maintained simultaneously in d dimensions. Assume $\mathrm{Tr}(\ldots)$ to be a meromorphic function of the dimension d, then show that (b) leads to

$$\begin{aligned} \mathrm{Tr}\left(\gamma_5\right) &= 0\,, \\ \mathrm{Tr}\left(\gamma_5\gamma_\mu\gamma_\nu\right) &= 0\,, \\ \mathrm{Tr}\left(\gamma_\mu\gamma_\nu\gamma_\rho\gamma_\sigma\gamma_5\right) &= 0\,. \end{aligned} \tag{4}$$

Exercise 4. ***An explicit $1/\epsilon^2$ pole***

Calculate the diagram in Figure 1 in dimensional regularisation to see that it has a pole of second order at $d = 4$.

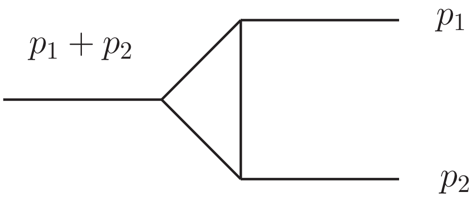

Figure 1: The one-loop vertex diagram in ϕ^3 theory.

Useful formulæ:

- The solid angle in d-dimensions is given by

$$\Omega_d = \int \mathrm{d}\Omega_d = \frac{2\pi^{d/2}}{\Gamma(d/2)} \,. \tag{5}$$

- The integral representation of the Beta function $B(a,b)$ is

$$B(a,b) = \frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)} = \int_0^1 \mathrm{d}x \, x^{a-1}(1-x)^{b-1} \,, \quad \Re a > 0 \,, \; \Re b > 0 \,. \tag{6}$$

- The Gauss hypergeometric function ${}_2F_1(a,b;c;z)$ is defined, for $|z| < 1$, by the series

$${}_2F_1(a,b;c;z) = \sum_{n=0}^{\infty} \frac{(a)_n (b)_n}{(c)_n} \frac{z^n}{n!} \,,$$

  where $(a)_n$ is the Pochhammer symbol, $(a)_n = \Gamma(a+n)/\Gamma(a)$.
  For $|\arg(1-z)| < \pi$, ${}_2F_1(a,b;c;z)$ admits the integral representation

$${}_2F_1(a,b;c;z) = \frac{\Gamma(c)}{\Gamma(c-b)\Gamma(b)} \int_0^1 \mathrm{d}t \, t^{b-1}(1-t)^{c-b-1}(1-z\,t)^{-a} \,, \quad \Re c > \Re b > 0 \,.$$