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Kern- und Teilchenphysik II
Lecture 5: The Higgs Mechanism

(adapted from the Handout of Prof. Mark Thomson)

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http://www.physik.uzh.ch/de/lehre/PHY213/FS2018.html

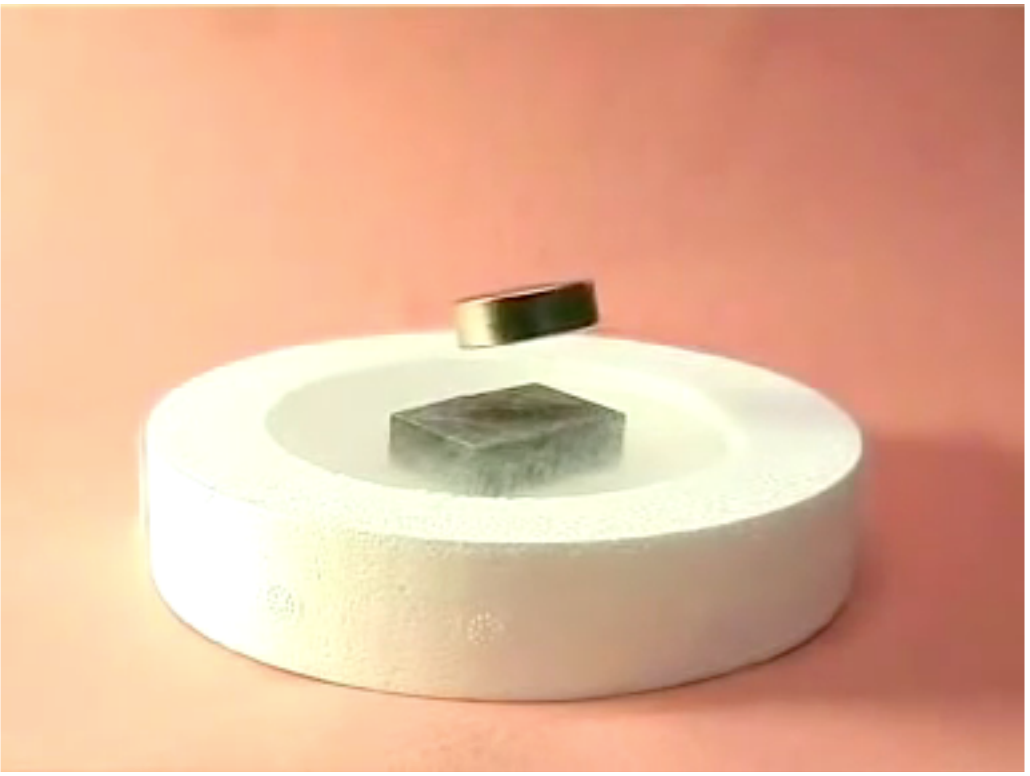
The Higgs Mechanism

★ We already introduced the ideas of gauge symmetries and electroweak unification. However, as it stands there is a small problem; this only works for massless gauge bosons. Introducing masses in any naïve way violates the underlying gauge symmetry.

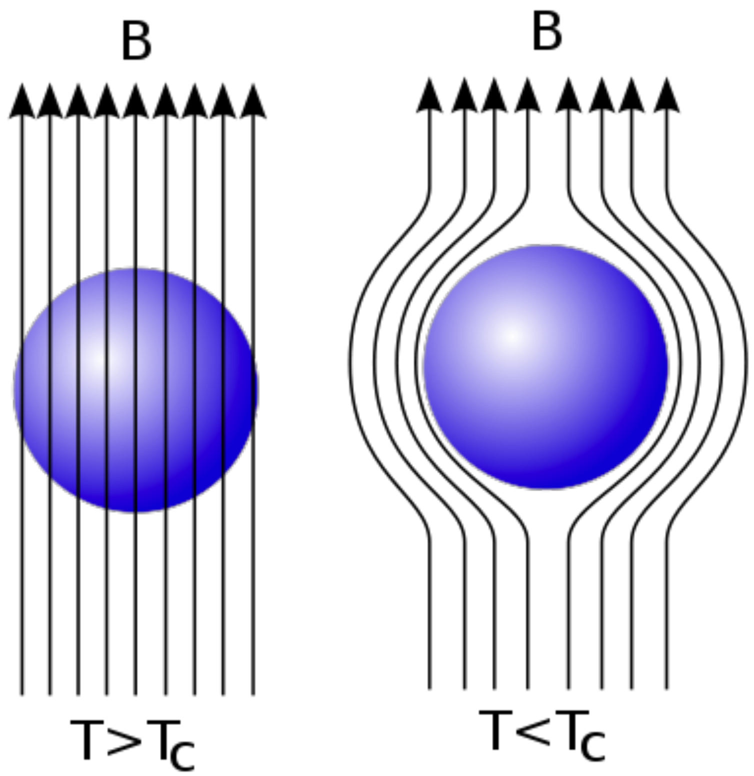
★ The Higgs mechanism provides a way of giving the gauge bosons mass

★ In this handout motivate the main idea behind the Higgs mechanism (however not possible to give a rigourous treatment outside of QFT). So resort to analogy:

★ Meissner Effect



Meissner Effect



Picture from wikipedia

★ **When superconductors are put in a magnetic field there is no magnetic field inside, field lines are excluded**

★ **This is described by the London equations**

- $\frac{\partial \vec{J}}{\partial t} = \frac{ne^2}{m}\vec{E}$
- $\vec{\nabla} \times \vec{J} = -\frac{ne^2}{m}\vec{B}$

- If we consider electron in a 'normal' conductor the average speed is given by $\vec{J} = ne\vec{v}_d$, where $\vec{v}_d$ is the drift velocity
- The drift velocity is given by $\vec{v}_d = \frac{e\vec{E}}{m}\tau$, where τ is the characteristic time between collisions and $\vec{E}$ is the electric field
- You have therefore $\vec{J} = \frac{ne^2\tau}{m}\vec{E}$, i.e. the current density is constant for a given $\vec{E}$

Meissner Effect

★ **Let's consider a perfect-conductor with "electrons" that are completely free and do not "scatter" (i.e. their free path is very large), in an electric field you'll have**

- $\vec{F} = e\vec{E}$, which implies $\frac{\partial \vec{v}}{\partial t} = \frac{e}{m}\vec{E}$, this gives the first London equation: $\frac{\partial \vec{J}}{\partial t} = \frac{ne^2}{m}\vec{E}$
- Using Farady's equation $\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$ you get $\vec{\nabla} \times \frac{\partial \vec{J}}{\partial t} = -\frac{ne^2}{m}\frac{\partial \vec{B}}{\partial t}$, which the Londons interpreted as the second equation: $\vec{\nabla} \times \vec{J} = -\frac{ne^2}{m}\vec{B}$

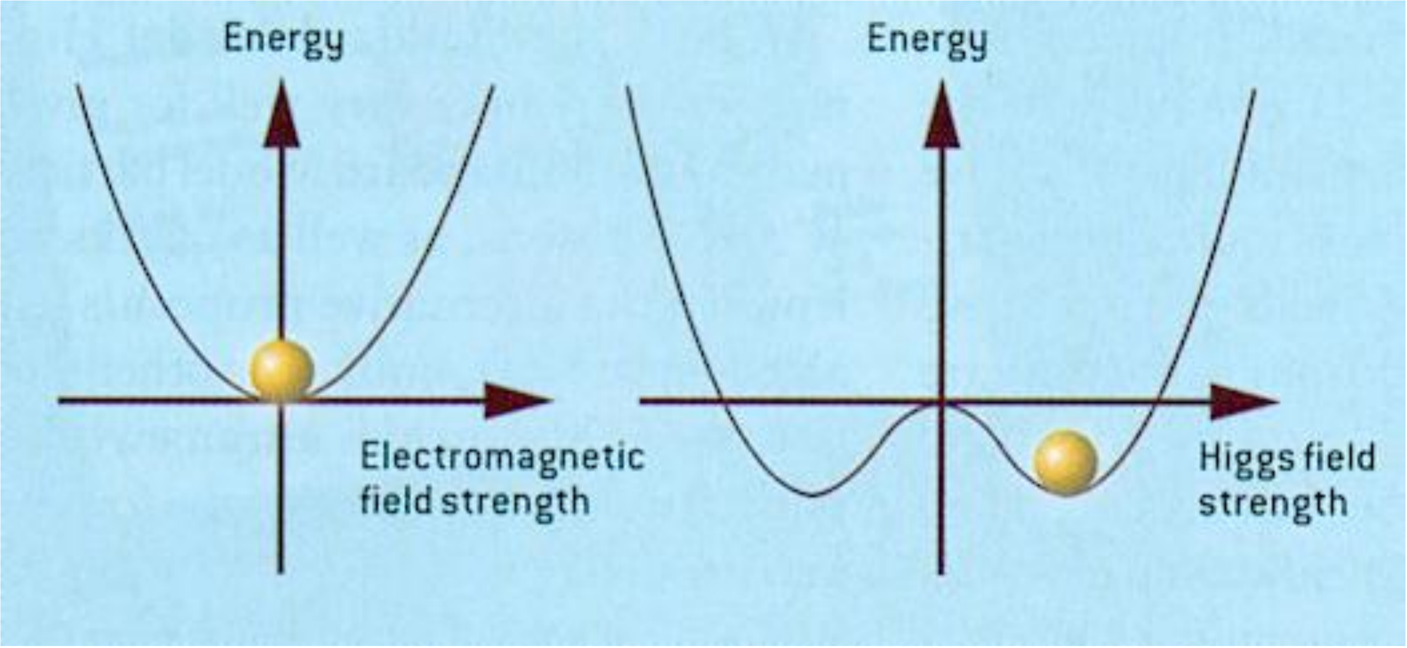
★ **The first equation says that the current density grows with time if you apply an electric field, the second that there cannot be a magnetic field inside the super-conductor (Meissner effect)**

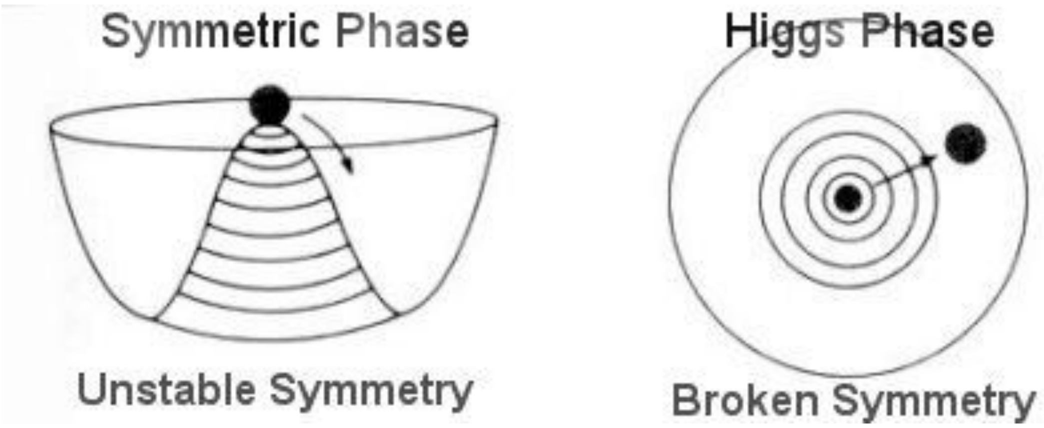
- Taking Ampere's low $\vec{\nabla} \times \vec{B} = \vec{J}$ and using second London equation we get: $\vec{\nabla}^2\vec{B} = -\frac{1}{\lambda^2}\vec{B}$
- The equation above describes the Meissner effect. In one dimension we have $\frac{\partial^2 B}{\partial x^2} = -\frac{1}{\lambda}B$, the solution is $B = B_0 e^{-\frac{x}{\lambda}}$, i.e. the magnetic field is exponentially decaying

Meissner Effect/Higgs Mechanism

- Let take the equation $\nabla^2 B + \frac{1}{\lambda^2} B = 0$, this is the static version of the Klein-Gordon equation $\partial_\mu \partial^\mu \psi + m^2 \psi = 0$ KTI Lecture9
- The photon is acquiring mass inside the superconductor!

Figure from www.universe-review.ca/





★ The Vacuum consists of many fields and particles are elementary excitations of the fields
★ The Lagrangian of the SM exhibit a symmetry that is U(1)xSU(2)xSU(3)
★ One of the field however has not the minimum at zero, it is still symmetric, but the minimum is shifted, this is the Higgs field
★This is known as spontaneous symmetry breaking, i.e. the symmetries of the Lagrangian are not symmetry of vacuum
★When we develop the perturbation theory close to the Higgs minimum we loose the symmetry, e.g. LH and RH are "attached together" like the Cooper pair in a superconductor

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Quantum Field and Lagrangian

★**Analytical Mechanics**

$$H = T(\dot{q}) + V(q), \qquad L = T(\dot{q}) - V(q)$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) - \frac{\partial L}{\partial q} = 0$$

← **Equation of motion** →

★**Field Theory**

$$\mathcal{L} \equiv \mathcal{L}(\psi,\ \partial_t\psi,\ \partial_x\psi,\ \partial_y\psi,\ \partial_z\psi)$$

$$\frac{\partial}{\partial\mu}\left(\frac{\partial\mathcal{L}}{\partial(\partial_\mu\psi)}\right) - \frac{\partial\mathcal{L}}{\partial\psi} = 0$$

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★ **Let's consider the lagrangian of a a scalar field** $\mathcal{L} = \partial_\mu\phi\partial^\mu\phi^*$ **applying the equation of motion we get KG equation for the massless case** $\partial_\mu\partial^\mu\phi = 0 \text{ or } (\partial_\mu\partial^\mu\phi^* = 0)$

★ **Let's consider the lagrangian** $\mathcal{L} = \partial_\mu\phi\partial^\mu\phi^* - m^2\phi\phi^*$ **applying the equation of motion we get KG equation for the massive case** $\partial_\mu\partial^\mu\phi + m^2\phi = 0 \text{ or } (\partial_\mu\partial^\mu\phi^* + m^2\phi^* = 0\ )$

Quantum Field and Lagrangian

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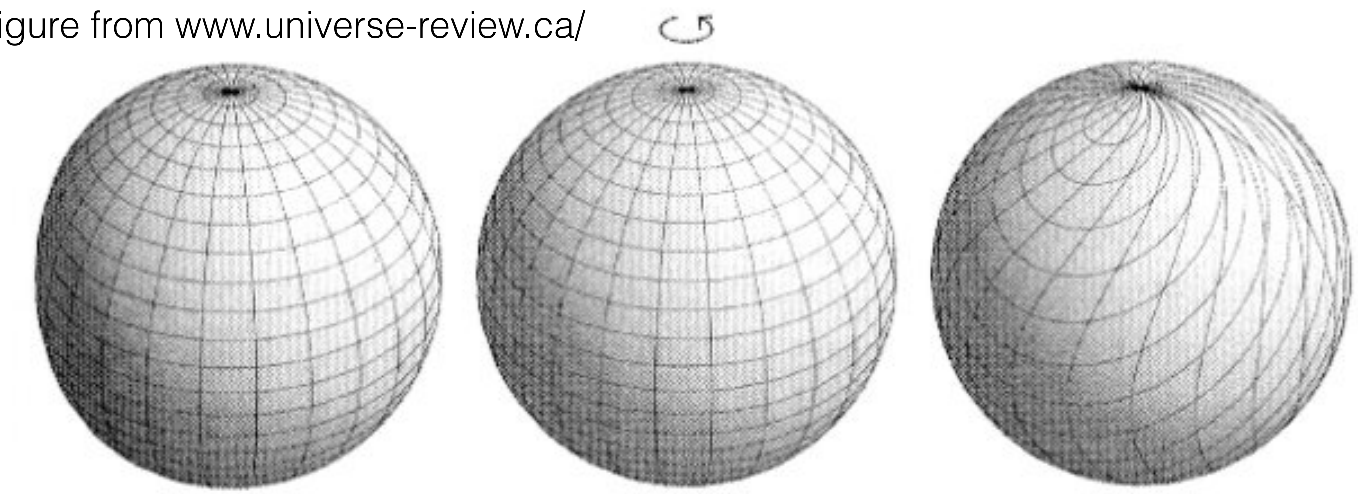
Kinetic term ↘   ↙ **Mass term**

$$\mathcal{L} = \partial_\mu\phi\partial^\mu\phi^* - m^2\phi\phi^*$$

Gauge invariance

★**The lagrangian of the complex ϕ is invariant by transformations** $\phi \to e^{i\alpha}\phi$ and $(\phi^* \to e^{-i\alpha}\phi^*)$

Figure from www.universe-review.ca/



Original Sphere | Global Transformation | Local Transformation

$$\mathcal{L} = \partial_\mu \phi \partial^\mu \phi^* - m^2 \phi\phi^*$$

★**If we want to be invariant under local gauge transformations** $\phi \to e^{i\alpha(x)}\phi$ and $(\phi^* \to e^{-i\alpha(x)}\phi^*)$

- $\partial_\mu \phi \to e^{i\alpha(x)}[\partial_\mu \phi + i\phi\partial_\mu]$
- $\partial_\mu \phi^* \to e^{-i\alpha(x)}[\partial_\mu \phi - i\phi\partial_\mu]$
- $\partial_\mu \phi^* \partial^\mu \phi \to \partial_\mu \phi^* \partial^\mu \phi + |\phi|^2 \partial_\mu \alpha(x) \partial^\mu \alpha(x)$
- $\delta\mathcal{L} = |\phi|^2 \partial_\mu \alpha(x) \partial^\mu \alpha(x)$

We need to cancel the term $\partial_\mu \alpha(x)$ in the derivative, i.e. we need a vector to dance these 4 components

Not local gauge invariant

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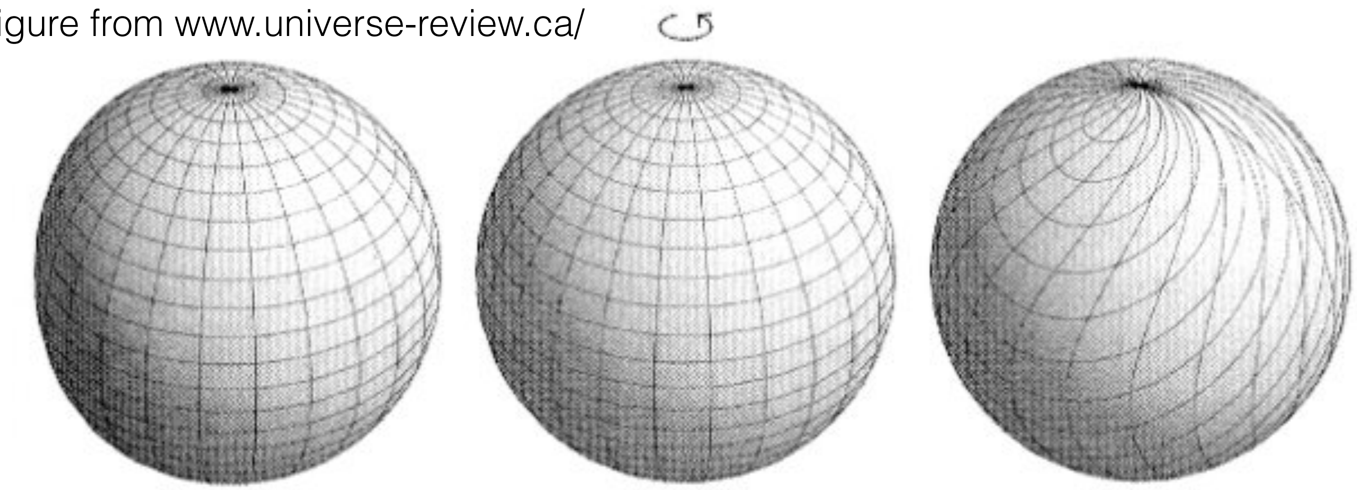
Gauge invariance

Figure from www.universe-review.ca/



★**If we want to have a local gauge invariance we need to introduce another field that transforms as**

- $A_\mu \rightarrow A_\mu + i\partial_\mu \alpha(x)$

★**The Lagrangian** $\mathcal{L} = [(\partial_\mu + iqA_\mu)\phi][(\partial^\mu - iqA^\mu)\phi^*]$ **is invariant under local gauge transformations**
★**We have inserted the field** A_μ**, so we need to add its own gauge invariant lagrangian**

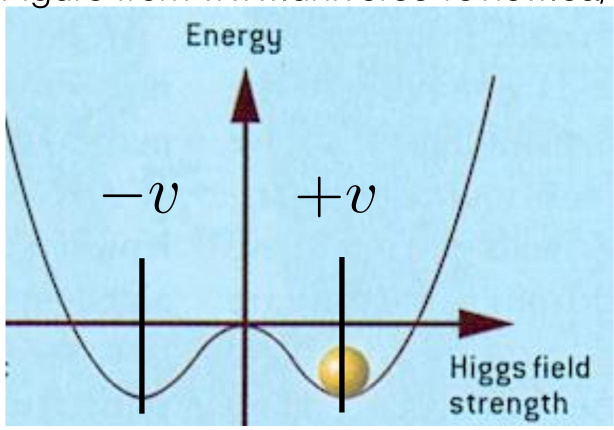
$$\mathcal{L} = [(\partial_\mu + iqA_\mu)\phi][(\partial^\mu - iqA^\mu)\phi^*] - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

Massless field for gauge invariance

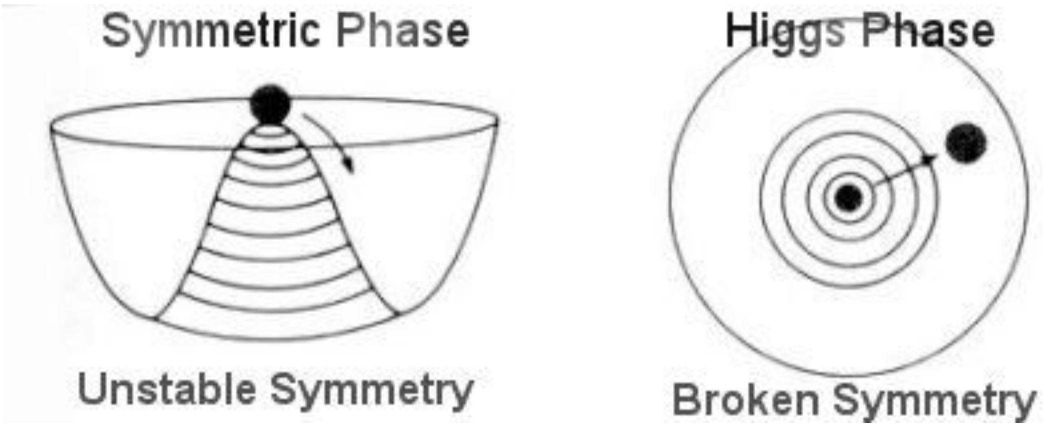
Higgs Mechanism (simplified)

Figure from www.universe-review.ca/



Let's take the Lagrangian with a mexican-hat scalar potential

$$\mathcal{L} = [(\partial_\mu + iqA_\mu)\phi][(\partial^\mu - iqA^\mu)\phi^*] - V(\phi,\phi^*) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$



$$V(\phi,\phi^*) = \alpha^2\phi\phi^* + \frac{1}{4}\lambda(\phi\phi^*)^2$$

Developing in the solution close to the point $\begin{pmatrix}\phi_1\\ \phi_2\end{pmatrix} = \begin{pmatrix}0\\ v+H(x)\end{pmatrix}$ **the Lagrangian becomes**

$$\mathcal{L} = \frac{1}{2}(\partial_\mu H\partial^\mu H + 2m^2H^2) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}q^2v^2A_\mu A^\mu + \ldots$$

Massive scalar field

Massive gauge boson
with mass qv, where v is the vev of the Higgs

$$v = \sqrt{\frac{-\alpha^2}{\lambda}}$$

Masses of the Fermions

★**The lagrangian for a massive Fermion is** $\mathcal{L} = i\overline{\psi}\gamma^\mu\partial_\mu\psi + m\overline{\psi}\psi$

Applying the equation of motion $\frac{\partial}{\partial\mu}\left(\frac{\partial\mathcal{L}}{\partial(\partial_\mu\psi)}\right) - \frac{\partial\mathcal{L}}{\partial\psi} = 0$

We get Dirac equation $i\gamma^\mu\partial_\mu\psi - m\psi = 0$

As said many times we can use the projection operator P_L and P_R to separate the LH and RH components $\psi = \psi_L + \psi_R$ **and we get**

$$\mathcal{L} = i\overline{\psi}_L\gamma^\mu\partial_\mu\psi_L + i\overline{\psi}_R\gamma^\mu\partial_\mu\psi_R + m(\overline{\psi}_L\psi_R + \overline{\psi}_R\psi_L)$$

Remember that for the commutation properties of the gamma matrices

$$\overline{\psi}_{L,R}\gamma^\mu\psi_{R,L} = 0 \text{ and } \overline{\psi}_{L,R}\psi_{L,R} = 0$$

Masses of the Fermions

★However, as we saw previously LH fermion form a doublet of SU(2), i.e. we should consider them as two component objects, while RH fermions are singlet

$$\mathcal{L} = i\overline{\chi}_L\gamma^\mu\partial_\mu\chi_L + i\overline{\psi}_R\gamma^\mu\partial_\mu\psi_R - m\overline{\chi}_L\psi_R$$

This term is a problem because it is not invariant under gauge transformation, i.e. it is not a scalar in the internal gauge space

$$\mathcal{L} = i\overline{\chi}_L\gamma^\mu\partial_\mu\chi_L + i\overline{\psi}_R\gamma^\mu\partial_\mu\psi_R - Y\overline{\chi}_L\phi\psi_R + \mathcal{L}_H + \ldots$$

We can make a gauge invariant term using the Higgs doublet, this is called Yukawa coupling. This is effectively a mass term coming from the interaction with the Higgs field.

The object Y is not a simple number is a non-diagonal matrix which makes eigenstate of flavour not eigenstate of mass.

History

★1960-1962 Anderson/Nambu discuss possible consequences of spontaneous symmetry breaking in particle physics

★A theory able to explain mass generation without breaking gauge theory was published by Englert, Higgs, Guralnik, Hagen and Kibble

★This mechanism is also known as Brout–Englert–Higgs mechanism, or Englert–Brout–Higgs–Guralnik–Hagen–Kibble mechanism

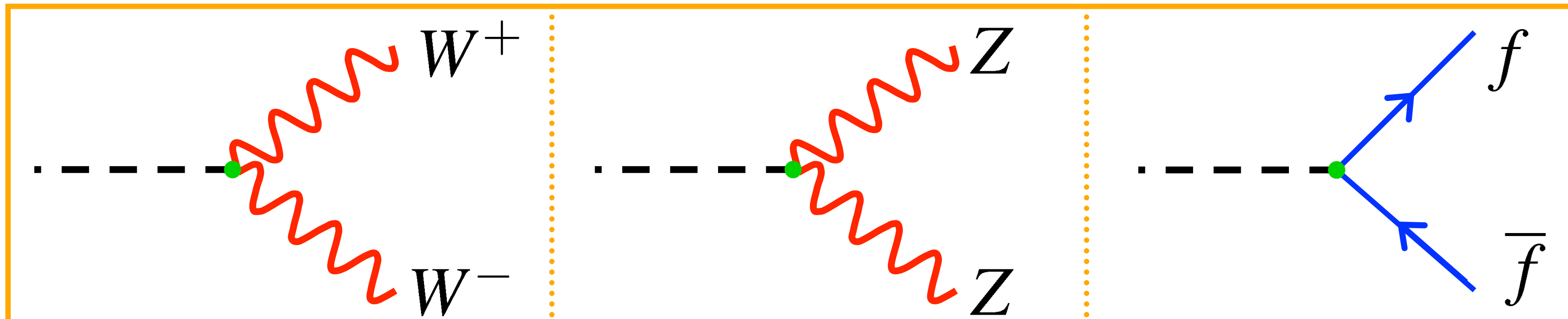
★Peter Higgs calls this mechanism ABEGHHK'tH mechanism acknowledging significant contributions from Anderson, Brout, Englert, Guralnik, Hagen, Higgs, Kibble and 't Hooft

The Higgs Mechanism

★ The Higgs mechanism results in absolute **predictions** for **masses of gauge bosons**

★ In the SM, fermion masses are also ascribed to interactions with the Higgs field
- however, here no prediction of the masses – **just put in by hand**

Feynman Vertex factors:



$ig_W m_W g^{\mu\nu}$	$ig_Z m_Z g^{\mu\nu}$	$-i\dfrac{g_W}{2m_W}m_f$

★ Within the SM of Electroweak unification with the Higgs mechanism:

Relations between standard model parameters

$$m_W = \left(\frac{\pi\alpha_{em}}{\sqrt{2}G_F}\right)^{\frac{1}{2}}\frac{1}{\sin\theta_W}$$

$$m_Z = \frac{m_W}{\cos\theta_W}$$

★ Hence, if you know **any three** of : $\alpha_{em}, G_F, m_W, m_Z, \sin\theta_W$ predict the other two.

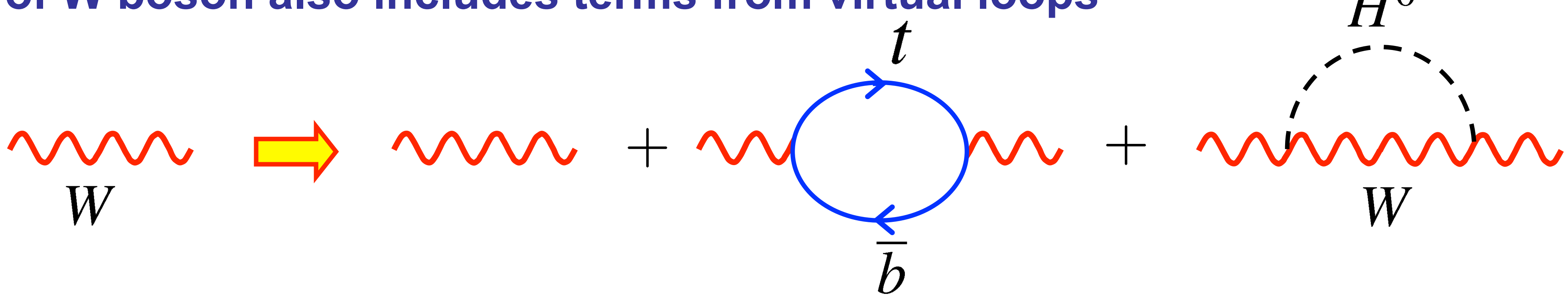
Precision tests of the SM

★ **From LEP and elsewhere have precise measurements – can test predictions of the Standard Model !**

- **e.g. predict:** $m_W = m_Z \cos\theta_W$ **measure** $m_Z = 91.1875 \pm 0.0021\,\mathrm{GeV}$, $\sin^2\theta_W = 0.23154 \pm 0.00016$
- **Therefore expect:** $m_W = 79.946 \pm 0.008\,\mathrm{GeV}$ **but measure** $m_W = 80.376 \pm 0.033\,\mathrm{GeV}$

★ **Close, but not quite right – but have only considered lowest order diagrams**

★ **Mass of W boson also includes terms from virtual loops**



$$m_W = m_W^0 + a m_t^2 + b \ln\left(\frac{m_H}{m_W}\right)$$

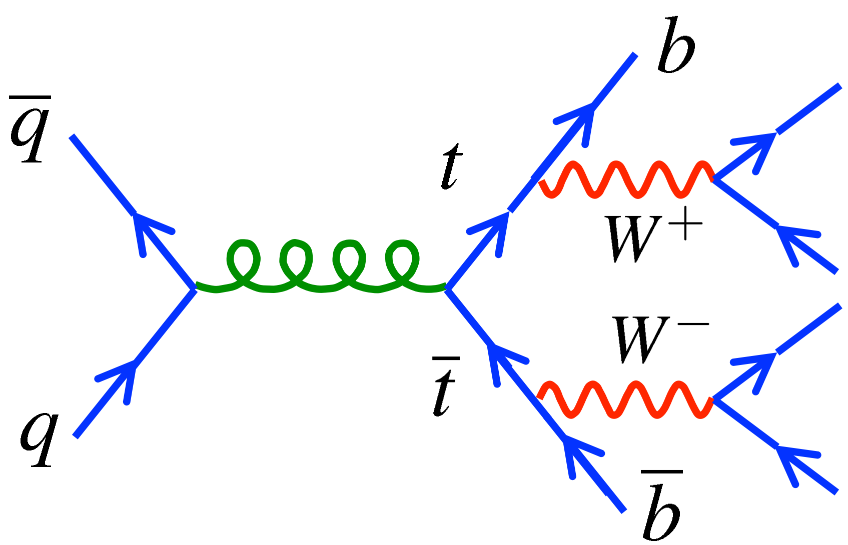
★ **Above "discrepancy" due to these virtual loops, i.e. by making very high precision measurements become sensitive to the masses of particles inside the virtual loops !**

The top quark

★ **From virtual loop corrections and precise LEP data can predict the top quark mass:**

$$m_t^{\mathrm{loop}} = 173 \pm 11\,\mathrm{GeV}$$

★ **In 1994 top quark observed at the Tevatron proton anti-proton collider at Fermilab – with the predicted mass !**



★ **The top quark almost exclusively decays to a bottom quark since**

$$|V_{tb}|^2 \gg |V_{td}|^2 + |V_{ts}|^2$$

★ **Complicated final state topologies:**

$$t\bar{t} \to b\bar{b}q\bar{q}q\bar{q} \to 6\text{ jets}$$

$$t\bar{t} \to b\bar{b}q\bar{q}\ell\nu \to 4\text{ jets} + \ell + \nu$$

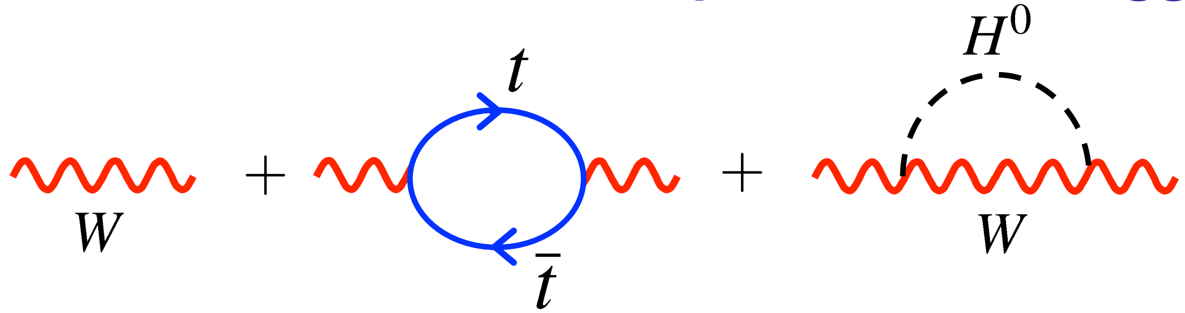
$$t\bar{t} \to b\bar{b}\ell\nu\ell\nu \to 2\text{ jets} + 2\ell + 2\nu$$

★ **Mass determined by direct reconstruction (see W boson mass)**

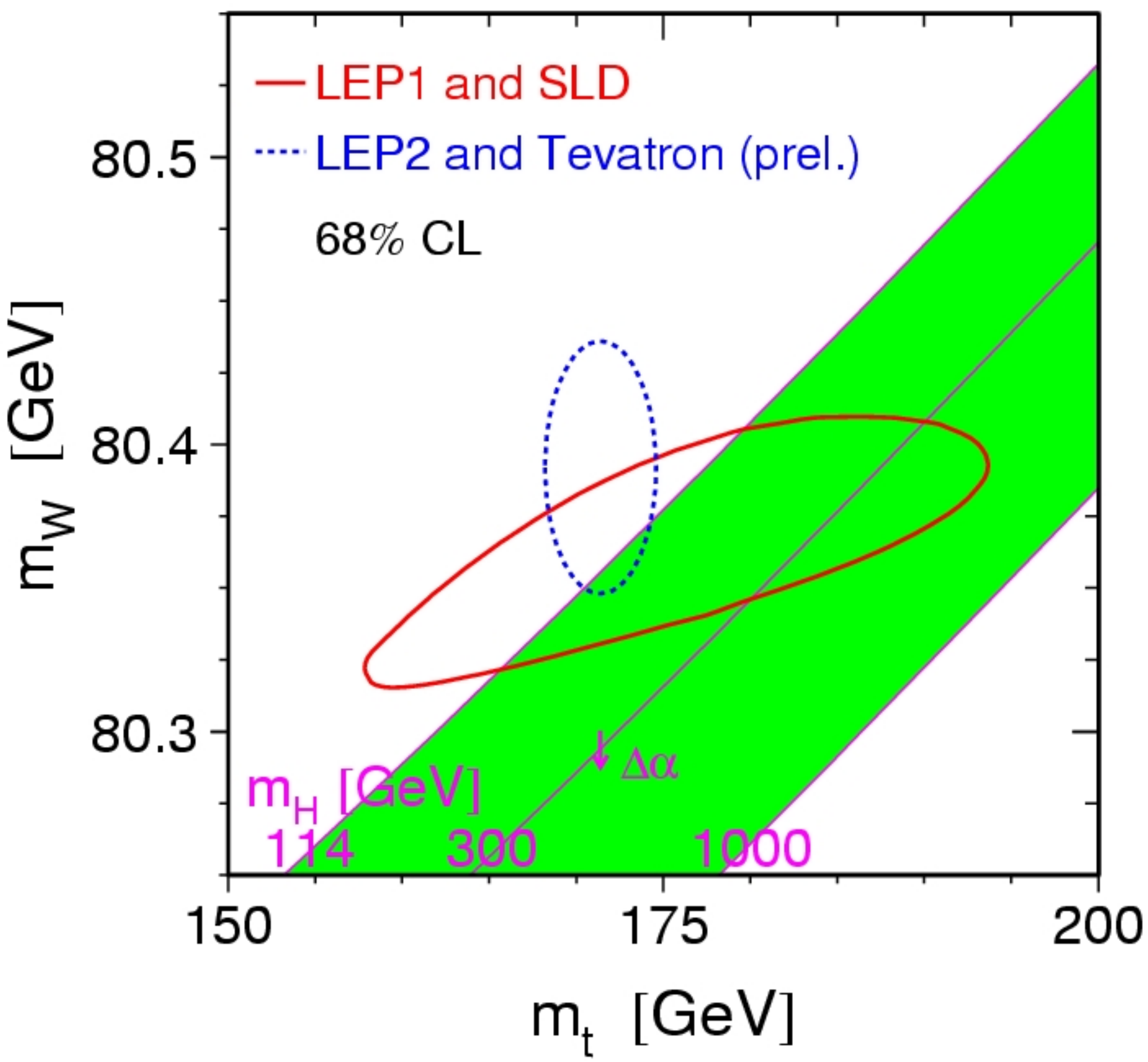
$$m_t^{\mathrm{meas}} = 174.2 \pm 3.3\,\mathrm{GeV}$$

The top quark

★ **But the W mass also depends on the Higgs mass (albeit only logarithmically)**



$$m_W = m_W^0 + a m_t^2 + b \ln\left(\frac{m_H}{m_W}\right)$$



★ **Measurements are sufficiently precise to have some sensitivity to the Higgs mass**

★ **Direct and indirect values of the top and W mass can be compared to prediction for different Higgs mass**

- **Direct:** W and top masses from direct reconstruction
- **Indirect:** from SM interpretation of Z mass, θ_W etc. and

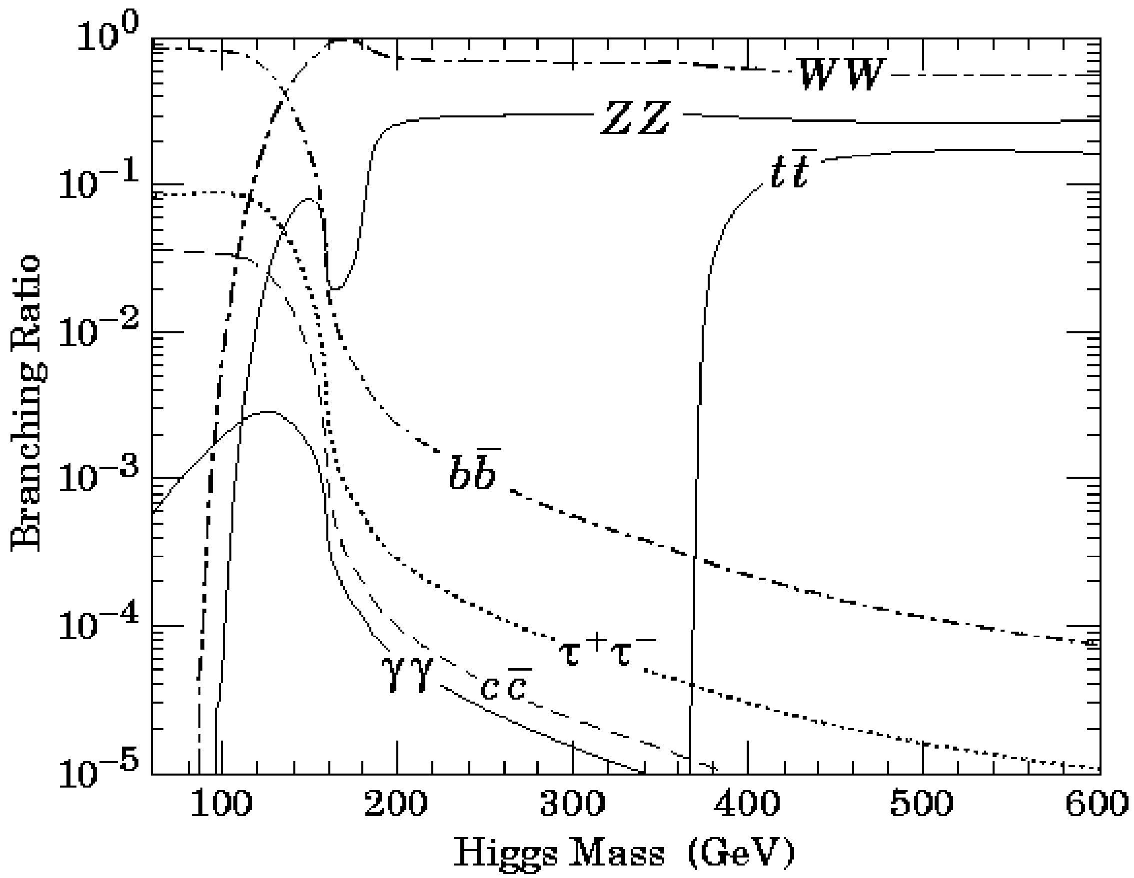
★ **Data favour a light Higgs:**

⇨ $m_H < 200\,\text{GeV}$

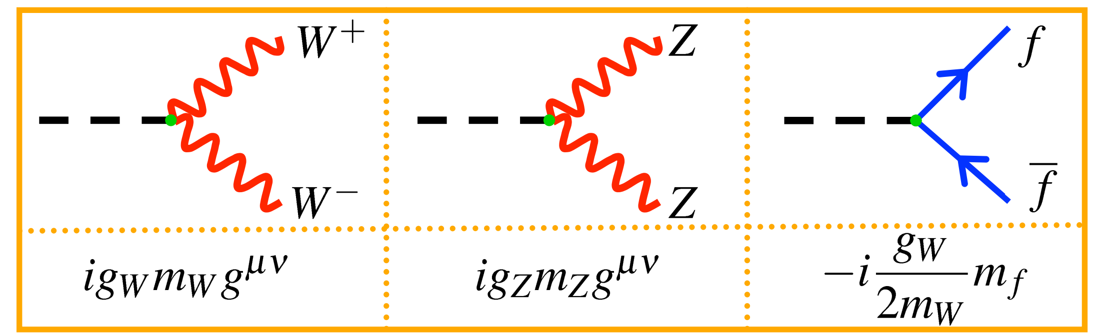
Hunting the Higgs

★ **The Higgs boson is an essential part of the Standard Model**

★ **Consider the search at LEP. Need to know how the Higgs decays**



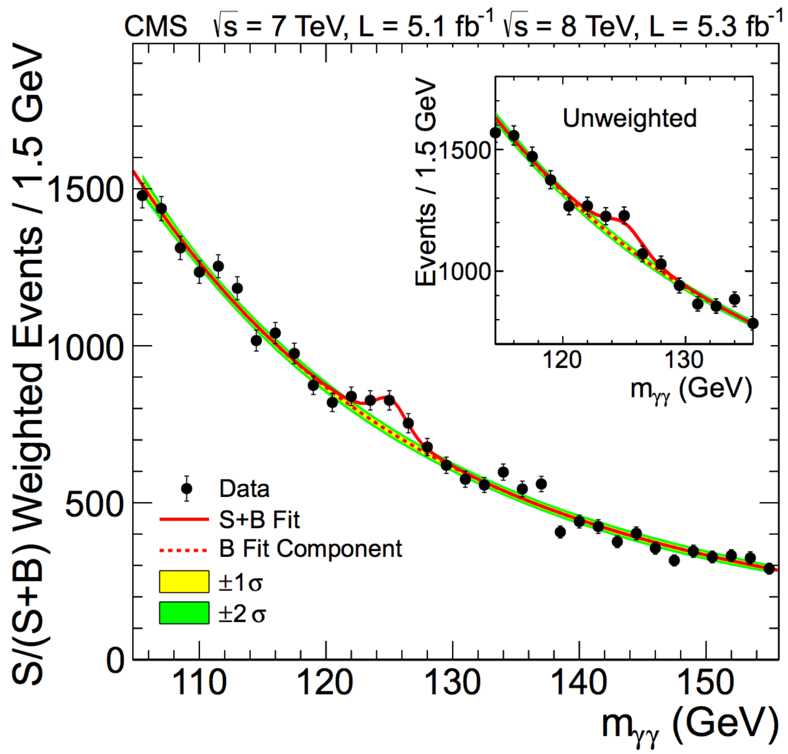
- **Higgs boson couplings proportional to mass**

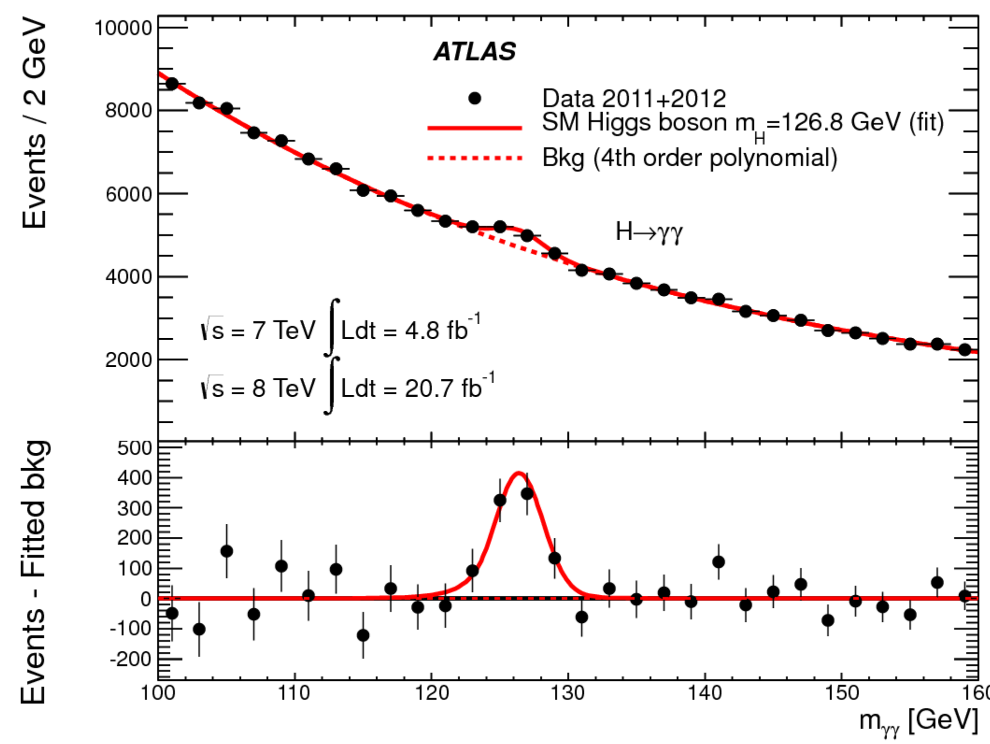


- **Higgs decays predominantly to heaviest particles which are energetically allowed**

$m_H < 2m_W$	**mainly**	$H^0 \to b\bar{b}$ **+ approx 10%** $H^0 \to \tau^+\tau^-$
$2m_W < m_H < 2m_t$	**almost entirely**	$H^0 \to W^+W^- \; + \; H^0 \to ZZ$
$m_H > 2m_t$	**either**	$H^0 \to W^+W^-,\; H^0 \to ZZ,\; H^0 \to t\bar{t}$

Higgs Discovery



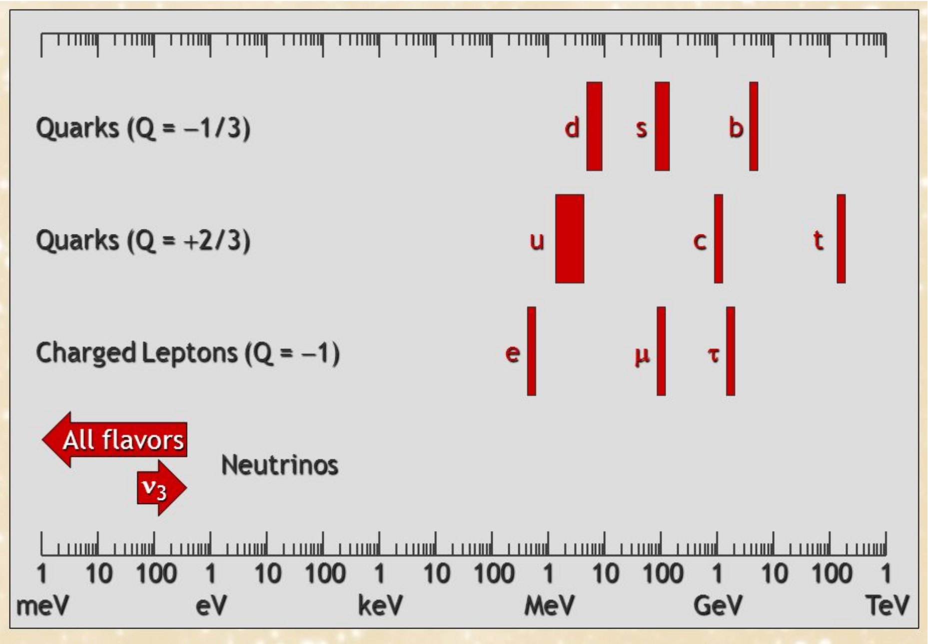


- The Higgs discovery of the Higgs boson form the ATLAS and CMS experiments at LHC showed that this picture is correct
- This is the last piece of puzzle of the SM
- More on the Higgs discovery will be discuss in a guest lecture

Neutrino masses

Neutrino masses pose a problem

- **Why neutrino masses are so small compared to the other fermions**
- **Does neutrino mass come from the Yukawa interaction as well? If that is the case what about RH neutrinos, do they exist?**
- **RH neutrinos are singles wrt the SM gauge group: no electric charge, do weak charge, no strong charge. This has several consequences since they can have a Majorana mass**
- **Majorana neutrinos are very important and their existence could have lots of implications for particle physics, but this discussion goes beyond the scope of this course (more on Majorana neutrinos will be discussed in a guest lecture)**



Picture by Georg Rafflet, Max Planck Institute