Formula Sheet

PHY111 – HS 2019

Physical constants

Gravitational constant	$G \approx 6.7 \times 10^{-11}\,\mathrm{Nm^2kg^{-2}}$
Free-fall acceleration	$g \approx 9.8\,\mathrm{ms^{-2}}$
Speed of sound in air	$v_{air} \approx 340\,\mathrm{m/s}$

Mathematics

$$\sqrt{2} = 1.4$$
$$\sqrt{1/2} = 0.70$$
$$\sqrt{3} = 1.7$$
$$\sqrt{1/3} = 0.57$$
$$\pi\,\mathrm{rad} = 180^\circ$$

Trigonometry

Trigonometry for common angles:

θ	$\sin\theta$	$\cos\theta$	$\tan\theta$
30°	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$
45°	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1
60°	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$

Small angle approximations:
$\sin x \approx x$
$\tan x \approx x$
$\cos x \approx 1 - \frac{x^2}{2}$

Gravity

Gravitational force $|F| = \frac{GMm}{r^2}$
Gravitational force (on Earth's surface) $F = mg$
Gravitational potential energy (on Earth's surface)
$U = mgh$

Mechanics

Linear motion, constant acceleration

$\boldsymbol{x}(t) = \boldsymbol{x}_0 + \boldsymbol{v}_0 t + \frac{1}{2}\boldsymbol{a}t^2$
$v_f^2 = v_0^2 + 2a\Delta x$
$\boldsymbol{v}(t) = \boldsymbol{v}_0 + \boldsymbol{a}t$
Momentum $\boldsymbol{p} = m\boldsymbol{v}$
$\sum \boldsymbol{F} = m\boldsymbol{a} = \frac{\Delta \boldsymbol{p}}{\Delta t}$
Impulse $I = \Delta p = \int F(t)dt$
Elastic force $F = -kx$
Friction force $F = \mu F_{\mathrm{N}}$
Center of mass $\boldsymbol{R}_{cm} = \frac{\sum_i m_i \boldsymbol{r}_i}{\sum_i m_i}$
Kinetic energy for linear motion $K = \frac{1}{2}mv^2$
Kinetic energy for a system of particles
$K = \frac{1}{2}\sum_i m_i v_{cm}^2 + \sum_i \frac{1}{2} m_i v_{i_{cm}}^2$
Work $W = \int \boldsymbol{F} \cdot d\boldsymbol{x} = \Delta K$
U = -W
Power $P = \boldsymbol{F} \cdot \boldsymbol{v}$

Circular motion

$\theta(t) = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$
$\omega(t) = \omega_0 + \alpha t$
Angular velocity $\omega = 2\pi\nu = \frac{d\theta}{dt} = \frac{v}{r} = \frac{2\pi}{T}$
Tangential velocity $\boldsymbol{v} = \boldsymbol{\omega} \times \boldsymbol{r}$
Centripetal acceleration $a_c = \frac{v^2}{r}$
Angular momentum $\boldsymbol{L} = \boldsymbol{r} \times \boldsymbol{p} = I\boldsymbol{\omega}$
Torque $\boldsymbol{\tau} = \boldsymbol{r} \times \boldsymbol{F}$
$\sum \boldsymbol{\tau} = I\boldsymbol{\alpha} = \frac{\Delta \boldsymbol{p}}{\Delta t} = \frac{d\boldsymbol{L}}{dt}$
Moment of inertia $I = \int r^2 dm$
Huygens-Steiner (parallel axis) theorem $I = I_{\mathrm{cm}} + mr^2$
Kinetic energy of a pure rotation $K = \frac{1}{2}I\omega^2$
Work for a pure rotation $W = \int_{\theta_1}^{\theta_2} \tau d\theta$
Power $P = \tau\omega$
Precession angular velocity $\omega_p = \frac{rmg}{I\omega}$

Simple Harmonic Oscillator

Simple harmonic oscillator $\frac{d^2\boldsymbol{x}}{dt^2} = -\omega^2\boldsymbol{x}(t)$
Solution to the equation of motion $x(t) = A\cos(\omega t + \phi)$
Damping force $F = -bv$
Damped oscillations $m\frac{d^2x(t)}{dt^2} = -b\frac{dx(t)}{dt} - kx(t)$
Damped frequency $\omega' = \omega_0\sqrt{1 - (\frac{b}{2m\omega_0})^2}$
Damping amplitude $A = A_0 e^{-\frac{b}{2m}t}$
Energy lost in a cycle $\frac{dE}{E} = -\frac{b}{m}dt$
Quality factor $Q = \frac{2\pi E}{|\Delta E|}$
Driving Force $F_{ext} = F_0\cos(\omega t)$
For driven oscillations
  Q-factor $Q = \frac{\omega_0}{\Delta\omega}$
  Amplitude $A = F_0/\sqrt{m^2(\omega_0^2 - \omega^2) + b^2\omega^2}$
  Phase $\tan\phi = b\omega/m(\omega_0^2 - \omega^2)$

Pseudo-Forces

In non-inertial reference frame $\boldsymbol{F}_p = -m\boldsymbol{a}$

Fluids

Pressure $P = \frac{F}{A}$
Bulk modulus $B = -\frac{P}{\Delta V/V}$
Continuity equation $I_v = Av = \text{constant}$
Bernoulli's equation $P + \rho gh + \frac{1}{2}\rho v^2 = \text{constant}$

Waves

Wave speed $v = \nu\lambda$
Wave equation $\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$
Speed of wave (for a string) $v = \sqrt{\frac{F}{\mu}}$
Harmonic wave $y(x,t) = A\sin(kx - \omega t)$
Wave number $k = \frac{2\pi}{\lambda}$
Energy density $\eta = \frac{E}{V}$
Intensity $I = \frac{\text{Power}}{A} = \eta v$
Doppler: source moving (u_s)
$f' = \frac{f_0}{1 \pm u_s/v}$, $-$: source approaching, $+$: source moving away
Doppler: receiver moving (u_r)
$f' = f_0(1 \pm \frac{u_r}{v})$, $-$: receiver moving away, $+$: receiver approaching
Standing waves $y(x,t) = A\cos(\omega t)\sin(kx)$

Interference

Constructive interference when $\Delta r = n\lambda$
Two-wave interference
$y_1 + y_2 = 2A\cos(\phi/2)\sin(kx - \omega t + \phi/2)$
Superimposition of standing waves
$y = \sum_n A_n\cos(\omega_n t + \phi_n)\sin(k_n x))$
Location of maxima for small angles $y = \frac{n\lambda D}{a}$
Combining two waves of different frequencies
$P_1 + P_2 = 2P_0\cos(\pi\Delta f t)\sin(2\pi f_{av} t)$

Sound waves

Speed of sound $v = \sqrt{\frac{B}{\rho}}$
Displacement $S = S_0\sin(kx - \omega t)$
Pressure change $P = P_0\sin(kx - \omega t - \pi/2)$
  with $P_0 = \rho\omega v S_0$
Intensity sound waves $I = \frac{1}{2}\frac{P_0^2}{\rho v}$
Sound intensity level $\beta = 10\log(\frac{I}{I_0})$
Young's modulus $Y = \frac{F/A}{\Delta L/L}$
Speed of sound in solids $v_s = \sqrt{\frac{Y}{\rho}}$
Shear modulus $M_s = \frac{F_s/a}{\tan\theta}$