Frühjahrssemester 2018
Prof. N. Serra

PHY213 - KT II
Exercise Sheet 1

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`http://www.physik.uzh.ch/de/lehre/PHY213/FS2018.html`

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Exercise 1: SU(3) Color algebra

Each QCD quark-gluon vertex contains a color matrix t^a_{ij}. The t^a_{ij} matrices are generators of the SU(3) color algebra, and they are hermitean and traceless (NOTE: $t^a_{ij} = \lambda^a_{ij}/2$ from page 16 Lecture 1).

When performing QCD loop calculations, it is necessary to sum over all color configurations of one diagram and thus a convenient way of evaluating color coefficients is needed. An example of this procedure can be seen when setting the normalization of the bare QCD coupling g_s.

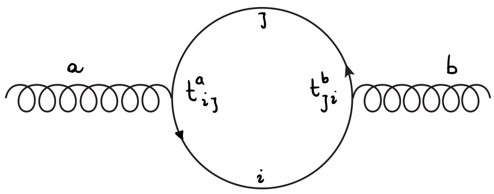

Figure 1: Quark loop in a gluon propagator

When considering the diagram in figure (1), the term $\mathrm{Tr}(t^a t^b) = t^a_{ij} t^b_{ji} = T_F \delta^{ab}$ has to be evaluated (summation over repeated indices is understood), and by convention T_F is set to 1/2.
Using su(3) generators properies, determine the value of the color coefficients C_F and C_A for the following loop diagrams

a) Quark self energy involving the product: $(t^a t^a)_{ij} = C_F \delta_{ij}$

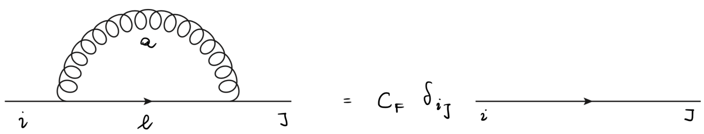

Figure 2: Quark self-energy diagram

b) Correction to the quark-gluon vertex where the product $\left(t^b t^a t^b\right)_{ij}$ appears.

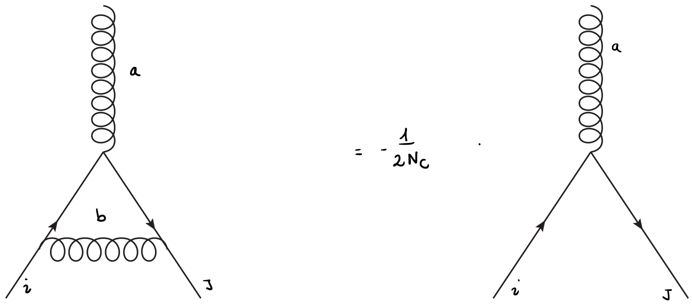

Figure 3: Correction to quark-gluon vertex

Exercise 2: W Vector boson polarization sums

The top quark decays to a bottom quark and a W^+ boson via the diagram in figure:

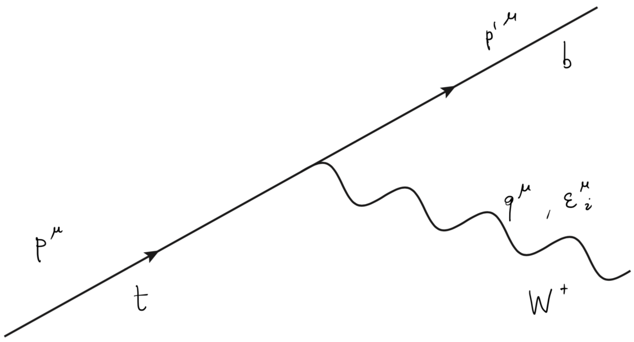

Figure 4: $t \to bW^+$ Feynman diagram

Where the momenta of the incoming and outcoming particles are labeled, as well as the polarization vector ϵ_i^μ for the W^+ boson.

a) Write down the matrix element for the decay

The differential decay width $d\Gamma$ for such decay is proportional to

$$d\Gamma \propto \frac{1}{2}\frac{1}{3}\sum_{i=1}^{3} \mathcal{M}^\mu \mathcal{M}^{*\nu} \epsilon^i_\mu \epsilon^{*i}_\nu \tag{1}$$

b) Can you guess the reason for the 1/2 and 1/3 prefactors?

c) Knowing that

$$\frac{1}{2}\frac{1}{3}\mathcal{M}^\mu \mathcal{M}^{*\nu} = \frac{g^2}{6}|V^{tb}|(p^\mu p'^\nu - (p \cdot p')g^{\mu\nu} + p^\nu p'^\mu) \tag{2}$$

Show that it makes no difference if using the polarization sums tensor

$$\Sigma^{\mu\nu} = \sum_{i=1}^{3} \epsilon^{i\mu}\epsilon^{*i\nu} = \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{m_W^2}\right) \tag{3}$$

where $g^{\mu\nu}$ is the metric tensor, or the explicit polarization vectors ϵ_μ^i (cfr. page 5 of Lecture 3), when summing over the W^+ boson polarizations.

d) EXTRA: Recalling the two body phase space factor $d\phi_2$ from exercise class 1 of KTI

$$d\phi_2 = \frac{1}{32\pi^2}\left(1 - \frac{m_W^2}{m_t^2}\right) d(cos\theta)d\phi \tag{4}$$

Determine the decay width for the process.

Exercise 3: Bilinear covariants under parity

Bilinear covariants are decompositions of $\bar{\psi}\Gamma\psi$ where ψ is a Dirac spinor, Γ is any combination of gamma matrices and $\bar{\psi}$ is the adjoint spinor (defined as $\bar{\psi} = \psi^\dagger\gamma^0$), that transform in a definite way under Lorentz transformation.

Using properties of gamma matrices show that:

a) The product $\bar{\psi}\gamma^\mu\psi$ transforms as a vector

b) The product $\bar{\psi}\gamma^5\psi$ transforms as a pseudoscalar