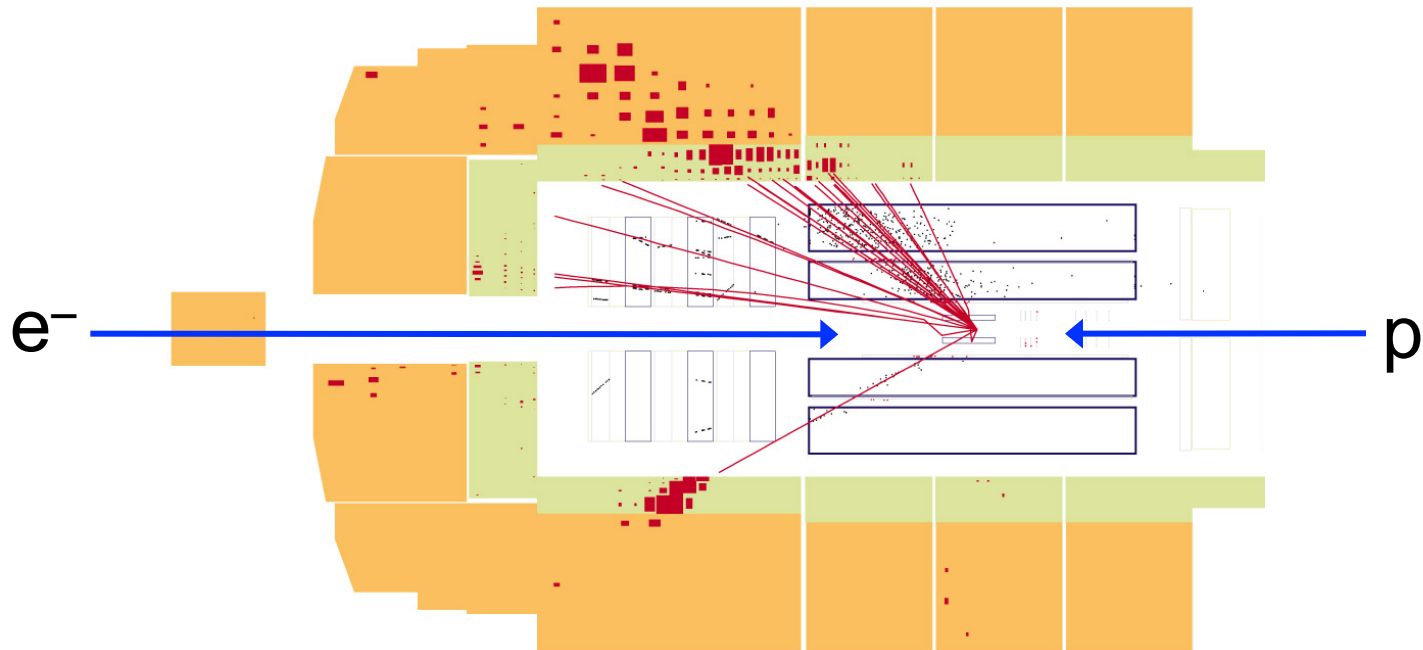# Particle Physics

Handout from Prof. Mar Thomson's lectures
Adapted to UZH by Prof. Canelli and Prof. Serra

e^- p

Handout 6 : Deep Inelastic Scattering

e⁻ p Elastic Scattering at Very High q^2

★ **At high q^2 the Rosenbluth expression for elastic scattering becomes**
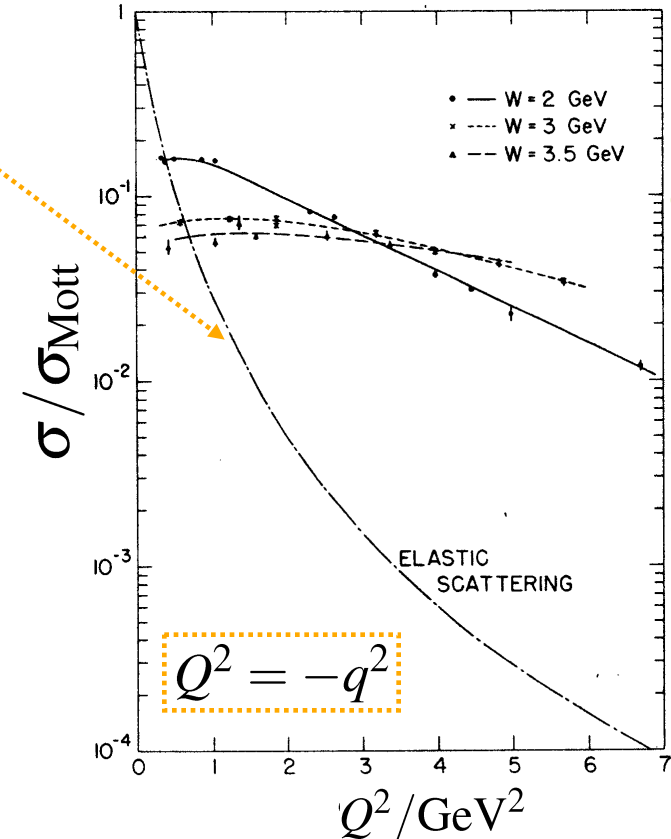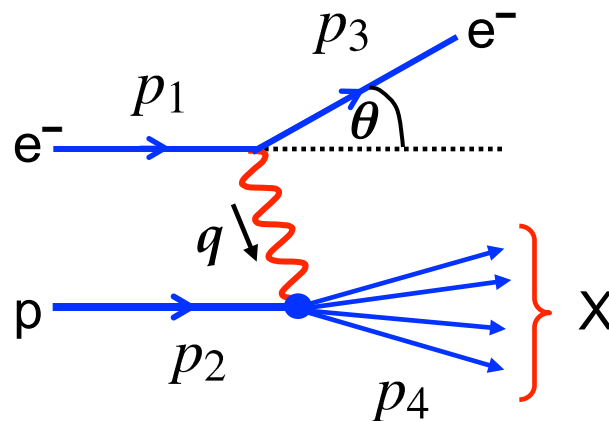
$$\left(\frac{d\sigma}{d\Omega}\right)_{elastic} = \frac{\alpha^2}{4E_1^2 \sin^4 \theta/2} \frac{E_3}{E_1} \left(\frac{q^2}{2M^2} G_M^2 \sin^2 \frac{\theta}{2}\right) \qquad \tau = -\frac{q^2}{4M^2} \gg 1$$

• **From e⁻ p elastic scattering, the proton magnetic form factor is**

$$G_M(q^2) \approx \frac{1}{(1+q^2/0.71\text{GeV}^2)^2} \qquad \Longrightarrow \qquad G_M(q^2) \propto q^{-4} \quad \text{at high } q^2$$

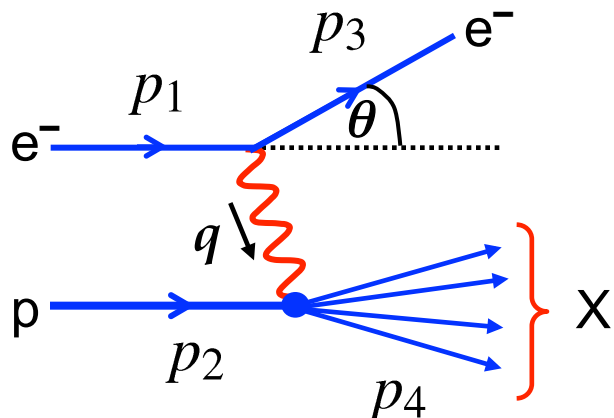$$\Longrightarrow \left(\frac{d\sigma}{d\Omega}\right)_{elastic} \propto q^{-6}$$

• **Due to the finite proton size, elastic scattering at high q^2 is unlikely and inelastic reactions where the proton breaks up dominate.**

Kinematics of Inelastic Scattering

- For inelastic scattering the mass of the final state hadronic system is no longer the proton mass, M

- The final state hadronic system must contain at least one **baryon** which implies the final state invariant mass $M_X > M$

$$M_X^2 = p_4^2 = (E_4^2 - |\vec{p}_4|^2)$$

✱ **For inelastic scattering introduce four new kinematic variables:** $\boxed{x, y, \nu, Q^2}$

★ **Define:** $\boxed{x \equiv \dfrac{Q^2}{2p_2.q}}$ **Bjorken x** **(Lorentz Invariant)**

where $\boxed{Q^2 \equiv -q^2}$ $\qquad Q^2 > 0$

- **Here** $\quad M_X^2 = p_4^2 = (q + p_2)^2 = -Q^2 + 2p_2.q + M^2$

$\Rightarrow \quad Q^2 = 2p_2.q + M^2 - M_X^2 \quad \Rightarrow \quad Q^2 \leq 2p_2.q$

Note: in many text books W is often used in place of M_X

hence $\boxed{0 < x < 1 \quad \textbf{inelastic}}$ $\qquad \boxed{x = 1 \qquad \textbf{elastic}}$ **Proton intact** $M_X = M$

★ **Define:** $\boxed{y \equiv \dfrac{p_2 \cdot q}{p_2 \cdot p_1}}$ **(Lorentz Invariant)**

• **In the Lab. Frame:**
$$p_1 = (E_1, 0, 0, E_1) \quad p_2 = (M, 0, 0, 0)$$
$$q = (E_1 - E_3, \vec{p}_1 - \vec{p}_3)$$

$$\Longrightarrow \quad y = \frac{M(E_1 - E_3)}{ME_1} = 1 - \frac{E_3}{E_1}$$

So y is the fractional energy loss of the incoming particle

$$\boxed{0 < y < 1}$$

• **In the C.o.M. Frame (neglecting the electron and proton masses):**
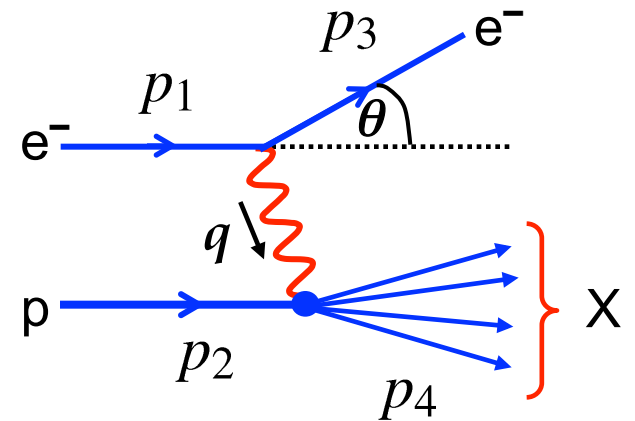$$p_1 = (E, 0, 0, E); \quad p_2 = (E, 0, 0, -E); \quad p_3 = (E, E\sin\theta^*, 0, E\cos\theta^*)$$

$$\Longrightarrow \quad y = \tfrac{1}{2}(1 - \cos\theta^*) \qquad \textbf{for} \quad E \gg M$$

★ **Finally Define:** $\boxed{\nu \equiv \dfrac{p_2 \cdot q}{M}}$ **(Lorentz Invariant)**
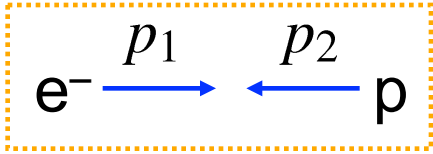
• **In the Lab. Frame:** $\nu = E_1 - E_3$

ν is the energy lost by the incoming particle

Relationships between Kinematic Variables

- **Can rewrite the new kinematic variables in terms of the squared centre-of-mass energy, s, for the electron-proton collision**

$e^- \xrightarrow{p_1} \quad \xleftarrow{p_2} p$

$$s = (p_1 + p_2)^2 = p_1^2 + p_2^2 + 2p_1.p_2 = 2p_1.p_2 + M^2 + \cancel{m_e^2}$$

Neglect mass of electron

$$2p_1.p_2 = s - M^2$$

- **For a fixed centre-of-mass energy, it can then be shown that the four kinematic variables**

$$Q^2 \equiv -q^2 \qquad x \equiv \frac{Q^2}{2p_2.q} \qquad y \equiv \frac{p_2.q}{p_2.p_1} \qquad \nu \equiv \frac{p_2.q}{M}$$

are not independent.

- **i.e. the scaling variables x and y can be expressed as**

$$x = \frac{Q^2}{2M\nu} \qquad\qquad y = \frac{2M}{s-M^2}\nu$$

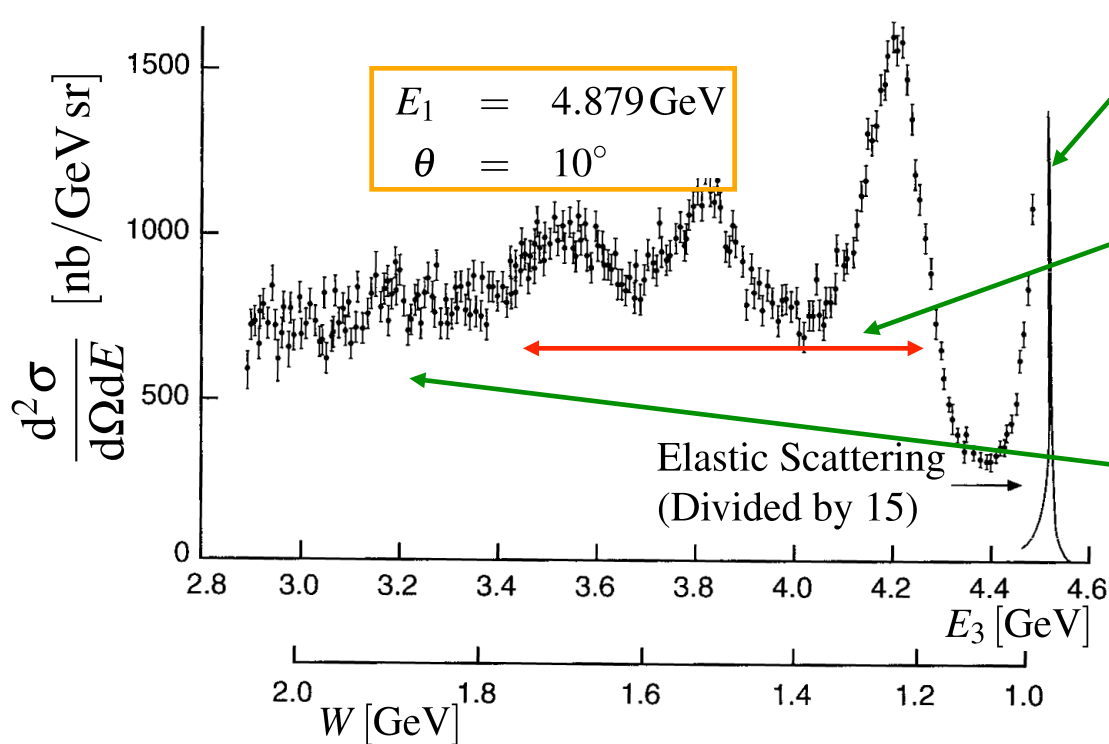*Note the simple relationship between y and ν*

and $\qquad xy = \dfrac{Q^2}{s-M^2} \qquad \Rightarrow \qquad Q^2 = (s-M^2)xy$

- **For a fixed centre of mass energy, the interaction kinematics are completely defined by any <u>two</u> of the above kinematic variables** (except y and ν)

- **For elastic scattering $(x = 1)$ there is only one independent variable. As we saw previously if you measure electron scattering angle know everything else.**

Inelastic Scattering

Example: **Scattering of 4.879 GeV electrons from protons at rest**
- **Place detector at 10° to beam and measure the energies of scattered e^-**
- **Kinematics fully determined from the electron energy and angle !**
- **e.g. for this energy and angle : the invariant mass of the final state hadronic system** $W^2 = M_X^2 = 10.06 - 2.03E_3$ **(try and show this)**
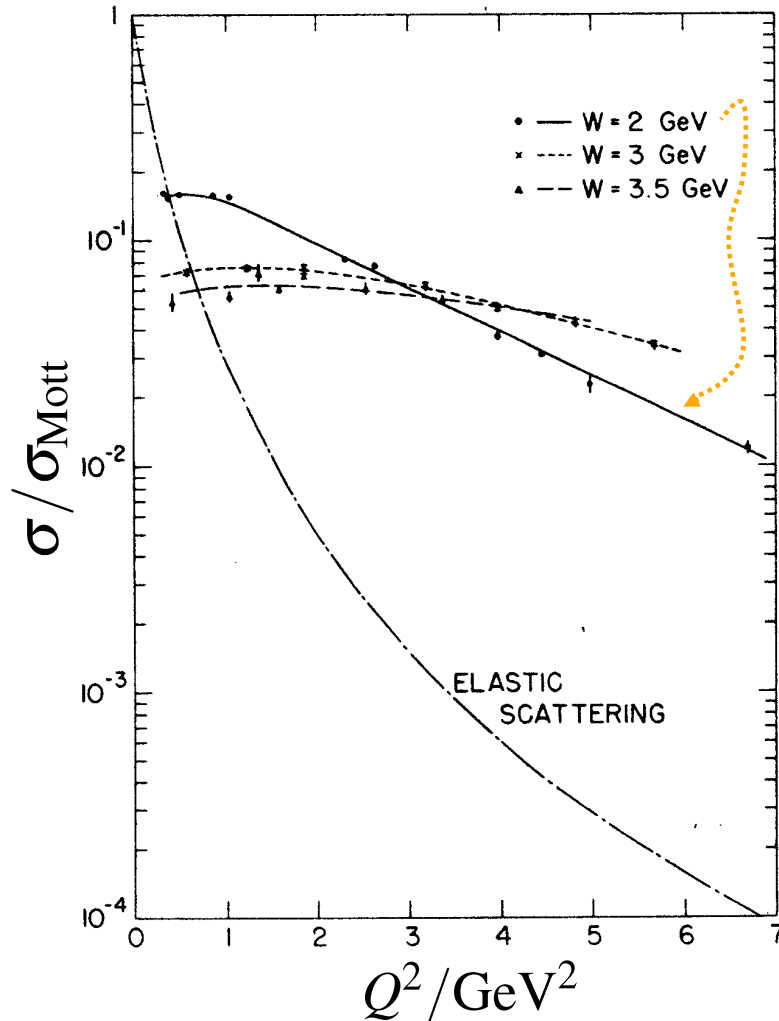

Elastic Scattering
proton remains intact
$$W = M$$

Inelastic Scattering
**produce "excited states"
of proton e.g.** $\Delta^+(1232)$
$$W = M_\Delta$$

Deep Inelastic Scattering
**proton breaks up resulting
in a many particle final state**

DIS = large W

Inelastic Cross Sections

- **Repeat experiments at different angles/beam energies and determine q^2 dependence of elastic and inelastic cross-sections**

- **Elastic scattering falls of rapidly with q^2 due to the proton not being point-like (i.e. form factors)**

- **Inelastic scattering cross sections only weakly dependent on q^2**

- **Deep Inelastic scattering cross sections almost independent of q^2 !**

  i.e. "Form factor" $\rightarrow$ 1

  ➡ **Scattering from point-like objects within the proton !**

Elastic → Inelastic Scattering

★ **Recall:** **Elastic scattering (Handout 5)**

- **Only one independent variable. In Lab. frame express differential cross section in terms of the electron scattering angle (Rosenbluth formula)**

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4E_1^2 \sin^4 \theta/2} \frac{E_3}{E_1} \left(\frac{G_E^2 + \tau G_M^2}{(1+\tau)} \cos^2 \frac{\theta}{2} + 2\tau G_M^2 \sin^2 \frac{\theta}{2} \right) \qquad \tau = \frac{Q^2}{4M^2}$$

Note: here the energy of the scattered electron is determined by the angle.

- **In terms of the Lorentz invariant kinematic variables can express this differential cross section in terms of Q^2** **(Q13 on examples sheet)**

$$\frac{d\sigma}{dQ^2} = \frac{4\pi\alpha^2}{Q^4} \left[\frac{G_E^2 + \tau G_M^2}{(1+\tau)} \left(1 - y - \frac{M^2 y^2}{Q^2} \right) + \frac{1}{2} y^2 G_M^2 \right]$$

which can be written as:

$$\frac{d\sigma}{dQ^2} = \frac{4\pi\alpha^2}{Q^4} \left[f_2(Q^2) \left(1 - y - \frac{M^2 y^2}{Q^2} \right) + \frac{1}{2} y^2 f_1(Q^2) \right]$$

★ **Inelastic scattering**

- **For Deep Inelastic Scattering have two independent variables. Therefore need a double differential cross section**

Deep Inelastic Scattering

★ **It can be shown that the most general Lorentz Invariant expression for** $e^-p \rightarrow e^-X$ **inelastic scattering (via a single exchanged photon is):**

$$\frac{\mathrm{d}^2\sigma}{\mathrm{d}x\mathrm{d}Q^2} = \frac{4\pi\alpha^2}{Q^4}\left[\left(1-y-\frac{M^2y^2}{Q^2}\right)\frac{F_2(x,Q^2)}{x}+y^2F_1(x,Q^2)\right] \quad \text{(1)}$$

(1) | **INELASTIC SCATTERING**

c.f. $$\frac{\mathrm{d}\sigma}{\mathrm{d}Q^2} = \frac{4\pi\alpha^2}{Q^4}\left[\left(1-y-\frac{M^2y^2}{Q^2}\right)f_2(Q^2)+\frac{1}{2}y^2f_1(Q^2)\right]$$

ELASTIC SCATTERING

We will soon see how this connects to the quark model of the proton

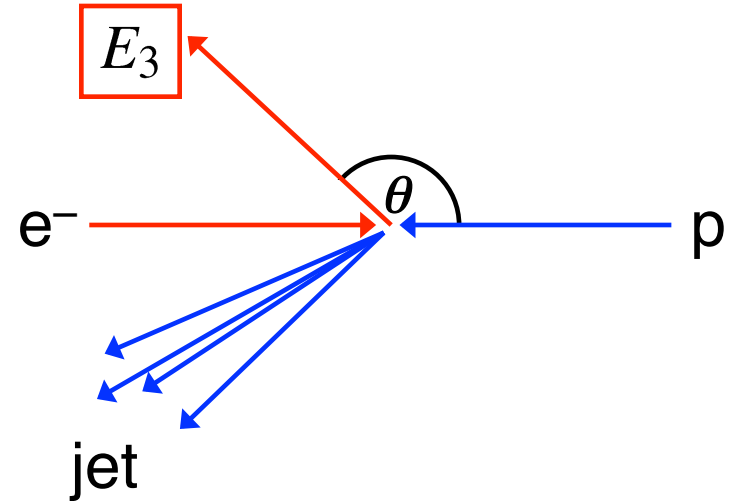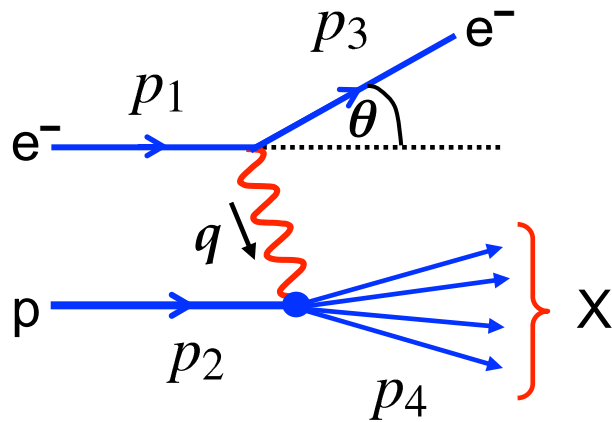• **NOTE: The form factors have been replaced by the STRUCTURE FUNCTIONS**

$$F_1(x,Q^2) \quad \text{and} \quad F_2(x,Q^2)$$

which are a function of x and Q^2: can not be interpreted as the Fourier transforms of the charge and magnetic moment distributions. We shall soon see that they describe the momentum distribution of the quarks within the proton

★ **In the limit of high energy (or more correctly $Q^2 \gg M^2y^2$) eqn. (1) becomes:**

$$\frac{\mathrm{d}^2\sigma}{\mathrm{d}x\mathrm{d}Q^2} = \frac{4\pi\alpha^2}{Q^4}\left[(1-y)\frac{F_2(x,Q^2)}{x}+y^2F_1(x,Q^2)\right] \quad \text{(2)}$$

- **In the Lab. frame it is convenient to express the cross section in terms of the angle, θ, and energy, E_3, of the scattered electron – experimentally well measured.**

$$Q^2 = 4E_1 E_3 \sin^2 \theta/2; \quad x = \frac{Q^2}{2M(E_1 - E_3)}; \quad y = 1 - \frac{E_3}{E_1}; \quad \nu = E_1 - E_3$$

- **In the Lab. frame, Equation (2) becomes:**   **(see examples sheet Q13)**

$$\frac{d^2\sigma}{dE_3 d\Omega} = \frac{\alpha^2}{4E_1^2 \sin^4 \theta/2} \left[\frac{1}{\nu} F_2(x, Q^2) \cos^2 \frac{\theta}{2} + \frac{2}{M} F_1(x, Q^2) \sin^2 \frac{\theta}{2} \right] \quad \text{(3)}$$
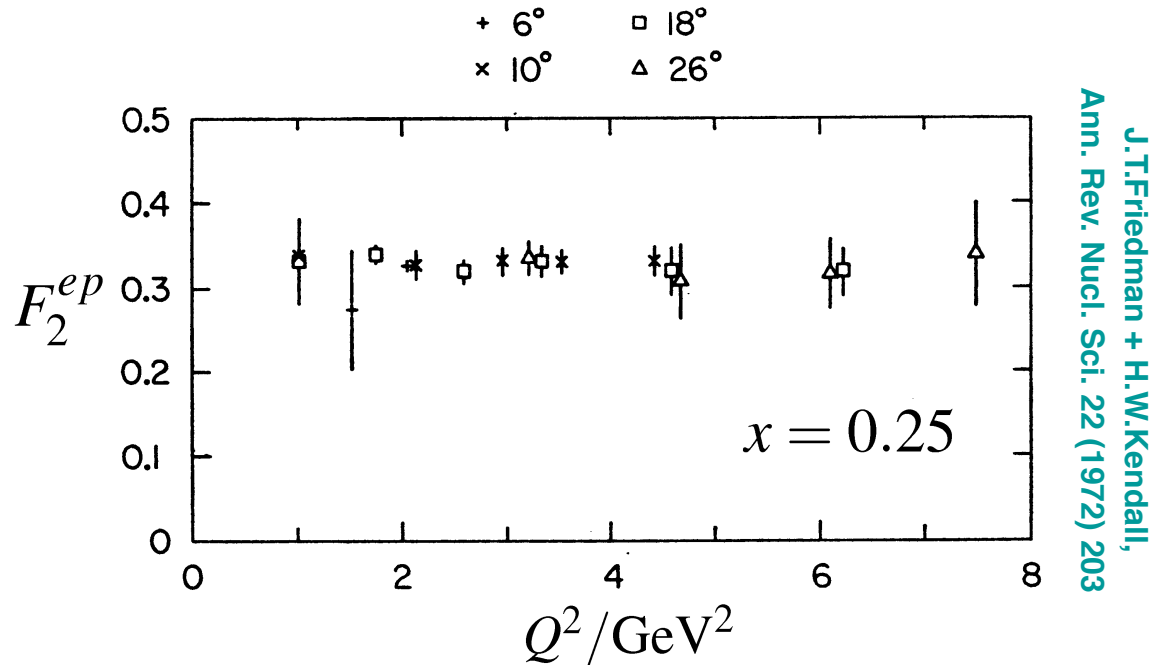
Electromagnetic Structure Function   **Pure Magnetic Structure Function**

Measuring the Structure Functions

★ **To determine $F_1(x, Q^2)$ and $F_2(x, Q^2)$ for a given x and Q^2 need measurements of the differential cross section at several different scattering angles and incoming electron beam energies (see Q13 on examples sheet)**

Example: electron-proton scattering F_2 vs. Q^2 at fixed x

J.T.Friedman + H.W.Kendall, Ann. Rev. Nucl. Sci. 22 (1972) 203

♦ **Experimentally it is observed that both F_1 and F_2 are (almost) independent of Q^2**

Bjorken Scaling and the Callan-Gross Relation

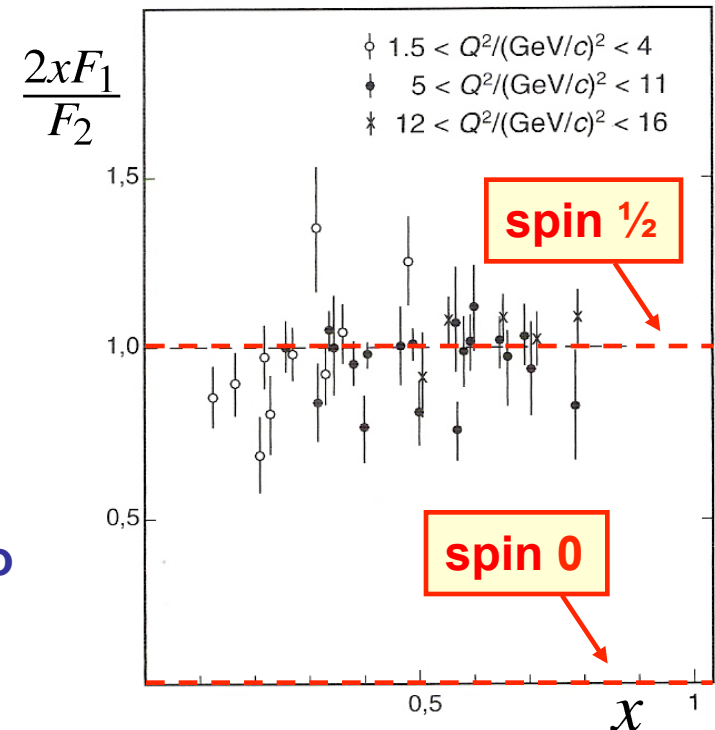★ **The near (see later) independence of the structure functions on Q^2 is known as Bjorken Scaling, i.e.**

$$F_1(x, Q^2) \to F_1(x) \qquad F_2(x, Q^2) \to F_2(x)$$

- **It is strongly suggestive of scattering from point-like constituents within the proton**

★ **It is also observed that** $F_1(x)$ **and** $F_2(x)$ **are not independent but satisfy the Callan-Gross relation**
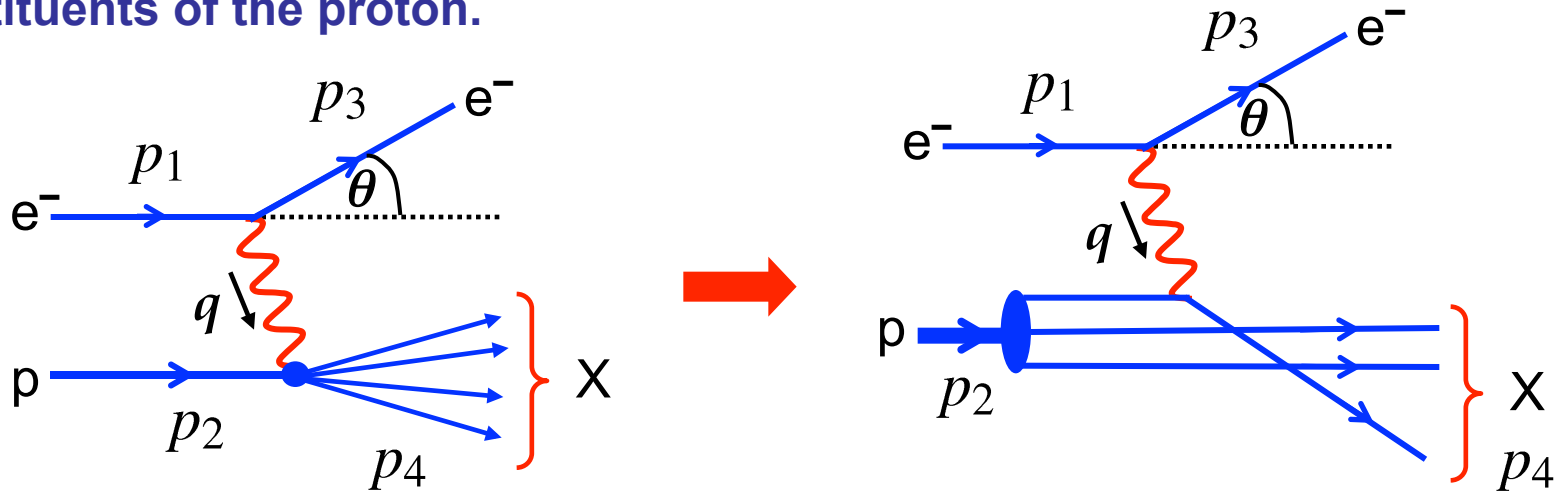
$$F_2(x) = 2xF_1(x)$$

- **As we shall soon see this is exactly what is expected for scattering from spin-half quarks.**

- **Note if quarks were spin zero particles we would expect the purely magnetic structure function to be zero, i.e.** $F_1(x) = 0$

$\dfrac{2xF_1}{F_2}$

$\phi\ \ 1.5 < Q^2/(\text{GeV}/c)^2 < 4$
$\bullet\ \ \ \ 5 < Q^2/(\text{GeV}/c)^2 < 11$
$\ast\ \ 12 < Q^2/(\text{GeV}/c)^2 < 16$

spin ½

spin 0

The Quark-Parton Model

- Before quarks and gluons were generally accepted Feynman proposed that the proton was made up of point-like constituents "**partons**"

- Both Bjorken Scaling and the Callan-Gross relationship can be explained by assuming that Deep Inelastic Scattering is dominated by the scattering of a single virtual photon from point-like spin-half constituents of the proton.

Scattering from a proton with structure functions → **Scattering from a point-like quark within the proton**

★ How do these two pictures of the interaction relate to each other?

- In the parton model the basic interaction is **ELASTIC** scattering from a "**quasi-free**" spin-½ quark in the proton, i.e. treat the quark as a free particle!

- The parton model is most easily formulated in a frame where the proton has very high energy, often referred to as the "**infinite momentum frame**", where we can neglect the proton mass and $p_2 = (E_2, 0, 0, E_2)$

- In this frame can also neglect the mass of the quark and any momentum transverse to the direction of the proton.

- Let the quark carry a fraction ξ of the proton's four-momentum.

- After the interaction the struck quark's four-momentum is $\xi p_2 + q$

$$(\xi p_2 + q)^2 = m_q^2 \approx 0 \quad \Rightarrow \quad \cancel{\xi^2 p_2^2} + q^2 + 2\xi p_2 . q = 0 \qquad (\xi^2 p_2^2 = m_q^2 \approx 0)$$

$$\Rightarrow \quad \xi = \frac{Q^2}{2 p_2 . q} = x$$

Bjorken x can be identified as the fraction of the proton momentum carried by the struck quark (in a frame where the proton has very high energy)
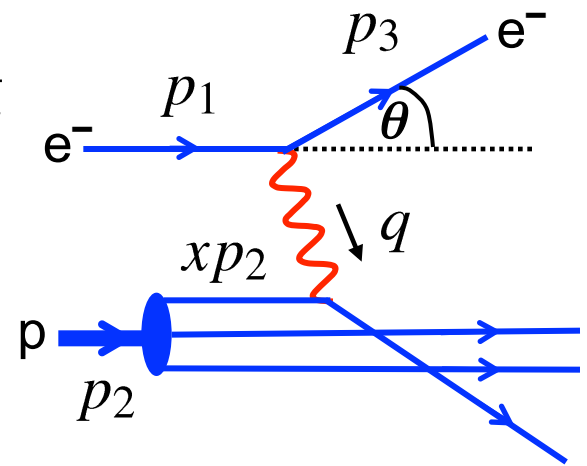
- **In terms of the proton momentum**

$$s = (p_1 + p_2)^2 \simeq 2p_1.p_2 \quad y = \frac{p_2.q}{p_2.p_1} \quad x = \frac{Q^2}{2p_2.q}$$

- **But for the underlying quark interaction**

$$s^q = (p_1 + xp_2)^2 = 2xp_1.p_2 = xs$$

$$y_q = \frac{p_q.q}{p_q.p_1} = \frac{xp_2.q}{xp_2.p_1} = y$$

$$x_q = 1 \quad \text{(elastic, i.e. assume quark does not break up)}$$

- **Previously derived the Lorentz Invariant cross section for** $e^-\mu^- \to e^-\mu^-$
  elastic scattering in the ultra-relativistic limit (handout 4 + Q10 on examples sheet).

  Now apply this to $e^-q \to e^-q$

$$\frac{d\sigma}{dq^2} = \frac{2\pi\alpha^2 e_q^2}{q^4}\left[1 + \left(1 + \frac{q^2}{s_q}\right)^2\right]$$

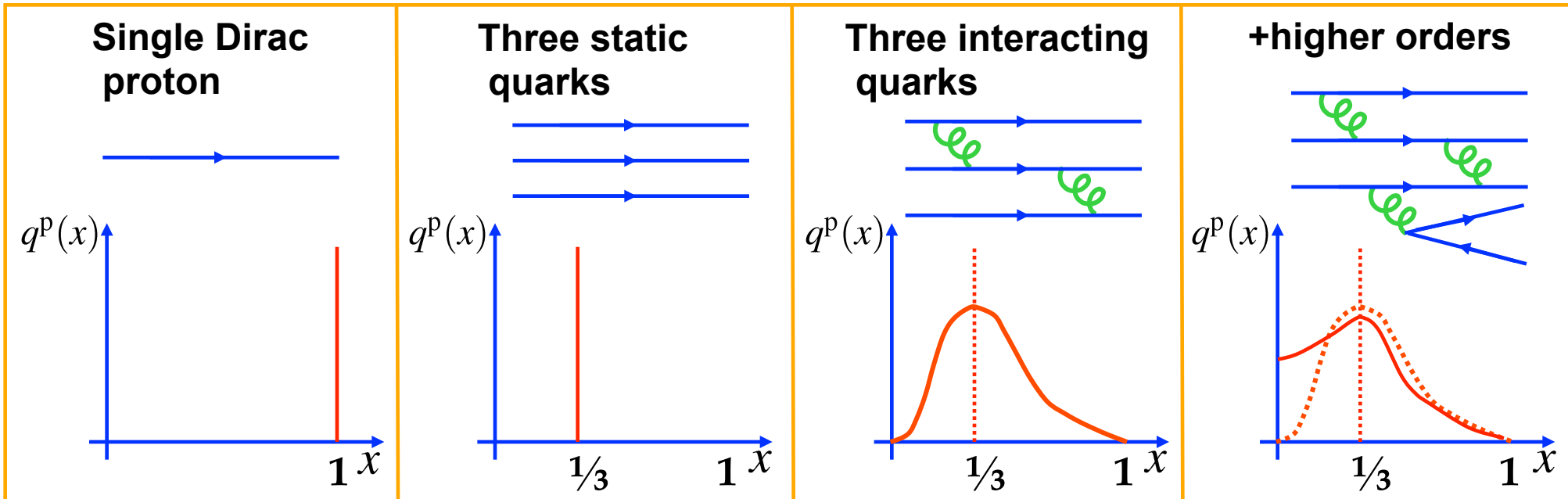e_q **is quark charge, i.e.**
$e_u = +2/3; \quad e_d = -1/3$

- **Using** $\quad -q^2 = Q^2 = (s_q - m^2)x_q y_q \quad \Longrightarrow \quad \dfrac{q^2}{s_q} = -y_q = -y$

$$\frac{d\sigma}{dQ^2} = \frac{2\pi\alpha^2 e_q^2}{Q^4}\left[1 + (1-y)^2\right]$$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}Q^2} = \frac{4\pi\alpha^2 e_q^2}{Q^4}\left[(1-y)+\frac{y^2}{2}\right]$$

(3)

★ **This is the expression for the differential cross-section for elastic** e⁻q
  scattering from a quark carrying a fraction x **of the proton momentum.**

 • **Now need to account for distribution of quark momenta within proton**

★ **Introduce parton distribution functions such that** $q^{\mathrm{p}}(x)\mathrm{d}x$ **is the number**
  of quarks of type q **within a proton with momenta between** $x \to x + \mathrm{d}x$

● **Expected form of the parton distribution function ?**

| Single Dirac proton | Three static quarks | Three interacting quarks | +higher orders |

★ **The cross section for scattering from a <u>particular quark type</u> within the proton which in the range $x \to x + \mathrm{d}x$ is**

$$\frac{\mathrm{d}^2\sigma}{\mathrm{d}Q^2} = \frac{4\pi\alpha^2}{Q^4}\left[(1-y)+\frac{y^2}{2}\right] \times e_q^2 q^{\mathrm{p}}(x)\mathrm{d}x$$

★ **Summing over all types of quark within the proton gives the expression for the electron-proton scattering cross section**

$$\frac{\mathrm{d}^2\sigma^{\mathrm{ep}}}{\mathrm{d}x\mathrm{d}Q^2} = \frac{4\pi\alpha^2}{Q^4}\left[(1-y)+\frac{y^2}{2}\right]\sum_q e_q^2 q^{\mathrm{p}}(x)$$

(5)

★ **Compare with the electron-proton scattering cross section in terms of structure functions (equation (2)):**

$$\frac{\mathrm{d}^2\sigma}{\mathrm{d}x\mathrm{d}Q^2} = \frac{4\pi\alpha^2}{Q^4}\left[(1-y)\frac{F_2(x,Q^2)}{x}+y^2 F_1(x,Q^2)\right]$$

(6)

★ **By comparing (5) and (6) obtain the parton model prediction for the structure functions in the general L.I. form for the differential cross section**

$$F_2^{\mathrm{p}}(x,Q^2) = 2xF_1^{\mathrm{p}}(x,Q^2) = x\sum_q e_q^2 q^{\mathrm{p}}(x)$$ ⟹ **Can relate measured structure functions to the underlying quark distributions**

The parton model predicts:

- **Bjorken Scaling** $F_1(x, Q^2) \rightarrow F_1(x)$ $F_2(x, Q^2) \rightarrow F_2(x)$
  - ✶ **Due to scattering from point-like particles within the proton**
- **Callan-Gross Relation** $F_2(x) = 2xF_1(x)$
  - ✶ **Due to scattering from spin half Dirac particles where the magnetic moment is directly related to the charge; hence the "electro-magnetic" and "pure magnetic" terms are fixed with respect to each other.**

★ **At present parton distributions cannot be calculated from QCD**
  - **Can't use perturbation theory due to large coupling constant**

★ **Measurements of the structure functions enable us to determine the parton distribution functions !**

★ **For electron-proton scattering we have:**

$$F_2^{\mathrm{p}}(x) = x \sum_q e_q^2 q^{\mathrm{p}}(x)$$

$\bar{q}$

- **Due to higher orders, the proton contains not only up and down quarks but also anti-up and anti-down quarks (will neglect the small contributions from heavier quarks)**

- **For electron-proton scattering have:**

$$F_2^{ep}(x) = x \sum_q e_q^2 q^p(x) = x\left(\frac{4}{9}u^p(x) + \frac{1}{9}d^p(x) + \frac{4}{9}\overline{u}^p(x) + \frac{1}{9}\overline{d}^p(x)\right)$$

- **For electron-neutron scattering have:**

$$F_2^{en}(x) = x \sum_q e_q^2 q^n(x) = x\left(\frac{4}{9}u^n(x) + \frac{1}{9}d^n(x) + \frac{4}{9}\overline{u}^n(x) + \frac{1}{9}\overline{d}^n(x)\right)$$

★**Now assume "isospin symmetry", i.e. that the neutron (ddu) is the same as a proton (uud) with up and down quarks interchanged, i.e.**

$$d^n(x) = u^p(x); \qquad u^n(x) = d^p(x)$$

and define the neutron distributions functions in terms of those of the proton

$$u(x) \equiv u^p(x) = d^n(x); \qquad d(x) \equiv d^p(x) = u^n(x)$$
$$\overline{u}(x) \equiv \overline{u}^p(x) = \overline{d}^n(x); \qquad \overline{d}(x) \equiv \overline{d}^p(x) = \overline{u}^n(x)$$

giving:

$$F_2^{ep}(x) = 2xF_1^{ep}(x) = x\left(\frac{4}{9}u(x) + \frac{1}{9}d(x) + \frac{4}{9}\overline{u}(x) + \frac{1}{9}\overline{d}(x)\right) \quad \textbf{(7)}$$

$$F_2^{en}(x) = 2xF_1^{en}(x) = x\left(\frac{4}{9}d(x) + \frac{1}{9}u(x) + \frac{4}{9}\overline{d}(x) + \frac{1}{9}\overline{u}(x)\right) \quad \textbf{(8)}$$

- **Integrating (7) and (8) :**

$$\int_0^1 F_2^{\text{ep}}(x)\mathrm{d}x = \int_0^1 x\left(\frac{4}{9}[u(x)+\bar{u}(x)] + \frac{1}{9}[d(x)+\bar{d}(x)]\right)\mathrm{d}x = \frac{4}{9}f_u + \frac{1}{9}f_d$$

$$\int_0^1 F_2^{\text{en}}(x)\mathrm{d}x = \int_0^1 x\left(\frac{4}{9}[d(x)+\bar{d}(x)] + \frac{1}{9}[u(x)+\bar{u}(x)]\right)\mathrm{d}x = \frac{4}{9}f_d + \frac{1}{9}f_u$$

$$\star \quad f_u = \int_0^1 [xu(x)+x\bar{u}(x)]\mathrm{d}x$$

is the fraction of the proton momentum carried by the up and anti-up quarks

<u>Experimentally</u>

$$\int F_2^{\text{ep}}(x)\mathrm{d}x \approx 0.18$$

$$\int F_2^{\text{en}}(x)\mathrm{d}x \approx 0.12$$

$$\Rightarrow \quad \boxed{f_u \approx 0.36 \qquad f_d \approx 0.18}$$

Area $= \frac{4}{9}f_u + \frac{1}{9}f_d$

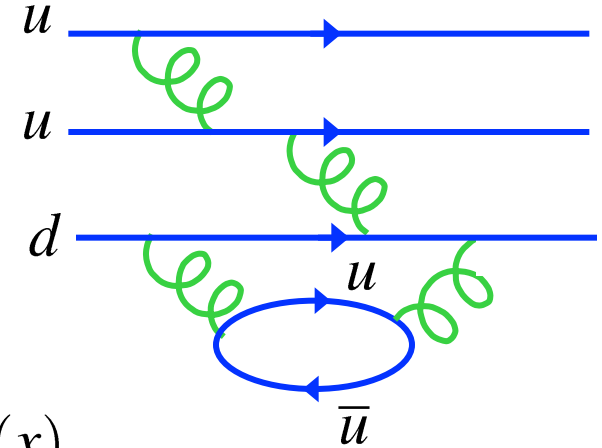$2\ (\text{GeV}/c)^2 < Q^2 < 18\ (\text{GeV}/c)^2$

$F_2^p(x, Q^2)$

★ **In the proton, as expected, the up quarks carry twice the momentum of the down quarks**

★ **The quarks carry just over 50 % of the total proton momentum. The rest is carried by gluons (which being neutral doesn't contribute to electron-nucleon scattering).**

Valence and Sea Quarks

- **As we are beginning to see the proton is complex…**

- **The parton distribution function** $u^p(x) = u(x)$
  includes contributions from the "valence"
  quarks and the virtual quarks produced by
  gluons: the "sea"

- **Resolving into valence and sea contributions:**

$$u(x) = u_V(x) + u_S(x) \qquad d(x) = d_V(x) + d_S(x)$$
$$\overline{u}(x) = \overline{u}_S(x) \qquad\qquad \overline{d}(x) = \overline{d}_S(x)$$

- **The proton contains two valence up quarks and one valence down quark**
  and would expect: $\displaystyle\int_0^1 u_V(x)\,\mathrm{d}x = 2 \qquad \int_0^1 d_V(x)\,\mathrm{d}x = 1$

- **But no *a priori* expectation for the total number of sea quarks !**

- **But sea quarks arise from gluon quark/anti-quark pair production and**
  with $m_u = m_d$ **it is reasonable to expect**

$$u_S(x) = d_S(x) = \overline{u}_S(x) = \overline{d}_S(x) = S(x)$$

- **With these relations (7) and (8) become**

$$F_2^{ep}(x) = x\left(\frac{4}{9}u_V(x) + \frac{1}{9}d_V(x) + \frac{10}{9}S(x)\right) \qquad F_2^{en}(x) = x\left(\frac{4}{9}d_V(x) + \frac{1}{9}u_V(x) + \frac{10}{9}S(x)\right)$$

Giving the ratio

$$\frac{F_2^{\text{en}}(x)}{F_2^{\text{ep}}(x)} = \frac{4d_V(x) + u_V(x) + 10S(x)}{4u_V(x) + d_V(x) + 10S(x)}$$

- The sea component arises from processes such as $g \to \overline{u}u$. Due to the $1/q^2$ dependence of the gluon propagator, much more likely to produce low energy gluons. Expect the sea to comprise of <u>low energy</u> $q/\overline{q}$

- Therefore at low x expect the sea to dominate:

$$\frac{F_2^{\text{en}}(x)}{F_2^{\text{ep}}(x)} \to 1 \quad \text{as} \quad x \to 0$$

Observed experimentally

- At high x expect the sea contribution to be small

$$\frac{F_2^{\text{en}}(x)}{F_2^{\text{ep}}(x)} \to \frac{4d_V(x) + u_V(x)}{4u_V(x) + d_V(x)} \quad \text{as} \quad x \to 1$$

Note: $u_V = 2d_V$ would give ratio 2/3 as $x \to 1$

Experimentally $F_2^{\text{en}}(x)/F_2^{\text{ep}}(x) \to 1/4$ as $x \to 1$

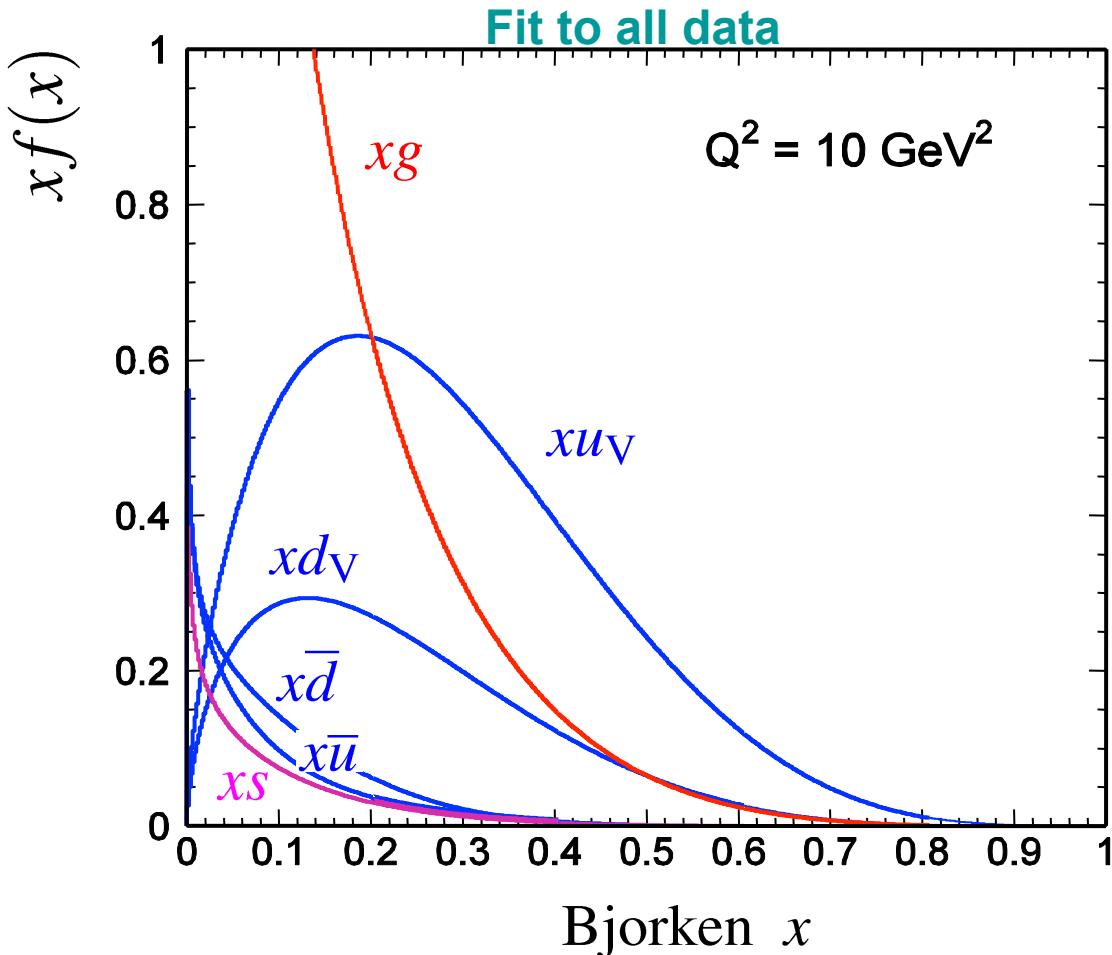$\Longrightarrow \quad d(x)/u(x) \to 0 \quad \text{as} \quad x \to 1$

This behaviour is not understood.

$\frac{F_2^{\text{en}}(x)}{F_2^{\text{ep}}(x)}$

$S(x)$ dominates

$u(x)$ dominates

x

Parton Distribution Functions

- **Ultimately the parton distribution functions are obtained from a fit to all experimental data including neutrino scattering (see handout 10)**

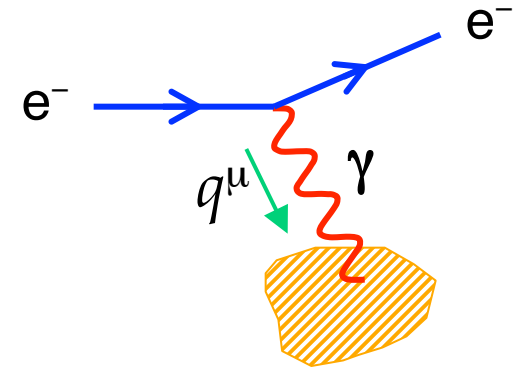 - Hadron-hadron collisions give information on gluon pdf $g(x)$

Fit to all data

$Q^2 = 10$ GeV^2

xg

xu_V

xd_V

$x\overline{d}$

$x\overline{u}$

xs

Bjorken x

Note:

- **Apart from at large x**
$$u_V(x) \approx 2d_V(x)$$
- **For $x < 0.2$ gluons dominate**
- **In fits to data assume**
$$u_s(x) = \overline{u}(x)$$
- $\overline{d}(x) > \overline{u}(x)$

  not understood – exclusion principle?
- **Small strange quark component $s(x)$**

(Try Question 12)

Scaling Violations

- In last 40 years, experiments have probed the proton with virtual photons of ever increasing energy

- Non-point like nature of the scattering becomes apparent when $\lambda_\gamma \sim$ size of scattering centre

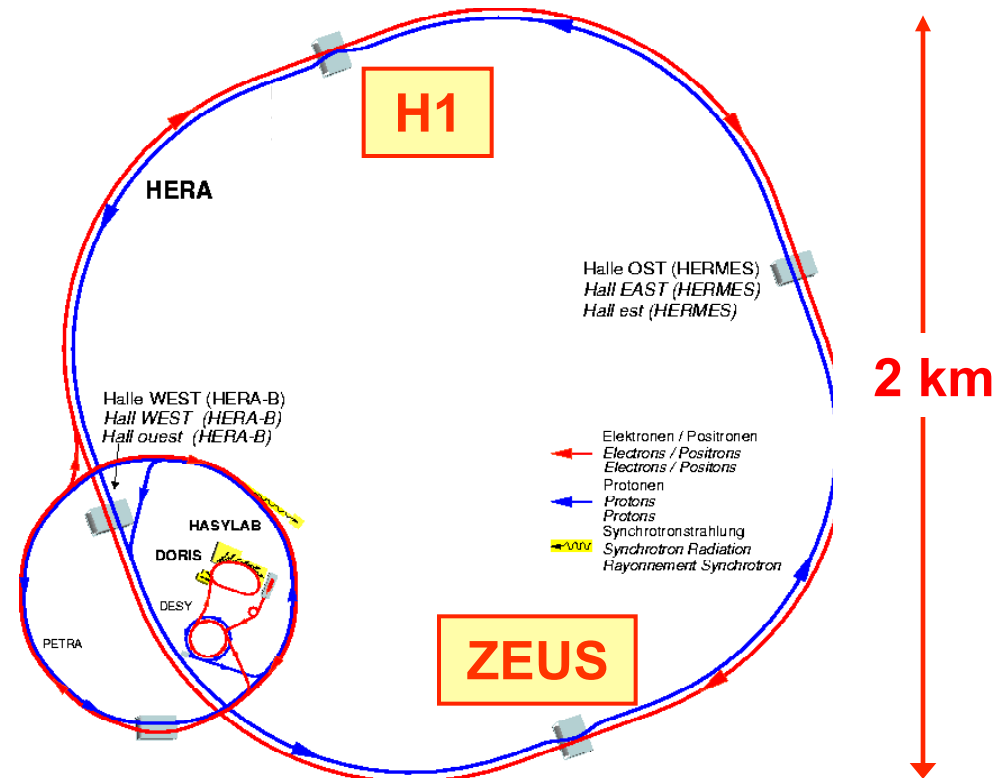$$\lambda_\gamma = \frac{h}{|\vec{q}|} \sim \frac{1\,\mathrm{GeV\,fm}}{|\vec{q}|\,(\mathrm{GeV})}$$

- Scattering from point-like quarks gives rise to **Bjorken scaling**: no q^2 cross section dependence

- IF quarks were not point-like, at high q^2 (when the wavelength of the virtual photon ~ size of quark) would observe rapid decrease in cross section with increasing q^2.

- To search for quark sub-structure want to go to highest q^2

HERA

HERA e±p Collider : 1991-2007

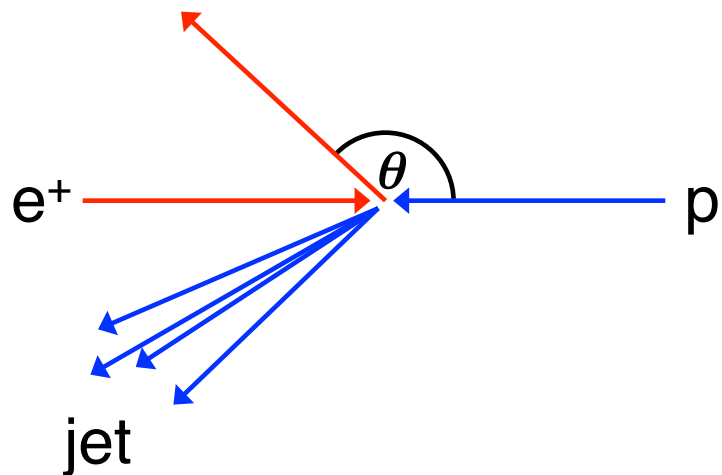★ **DESY (Deutsches Elektronen-Synchroton) Laboratory, Hamburg, Germany**

$e^{\pm}$ **27.5 GeV** → ← **820 GeV** p

$\sqrt{s} = 300$ GeV

H1

ZEUS

2 km

★ **Two large experiments : H1 and ZEUS**

★ **Probe proton at very high Q^2 and very low x**

Example of a High Q² Event in H1

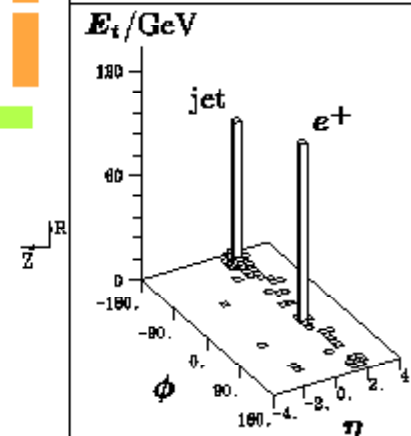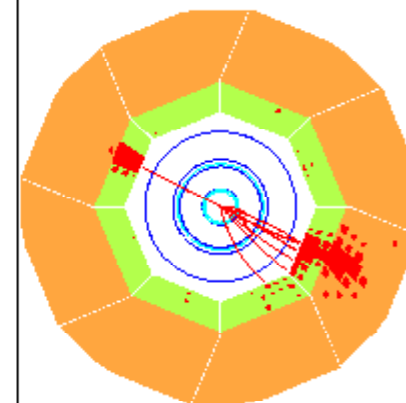★ **Event kinematics determined from electron angle and energy**

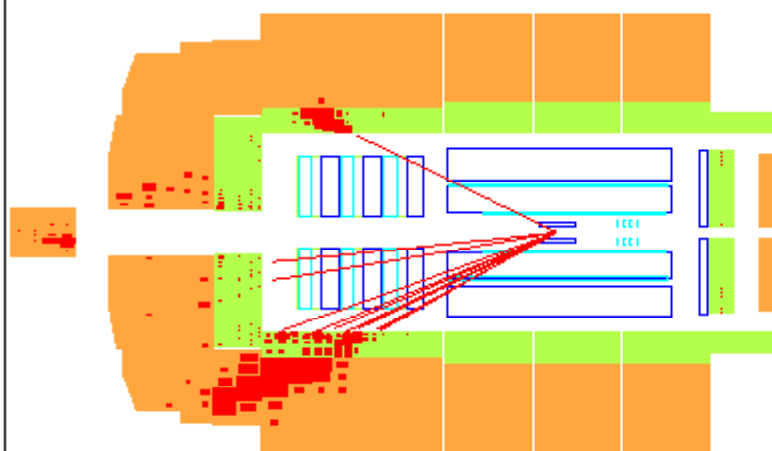★ **Also measure hadronic system (although not as precisely) – gives some redundancy**

H1   Run 122145   Event 69506   Date 19/09/1995

$Q^2 = 25030 \ \mathrm{GeV^2}, \quad y = 0.56, \quad M = 211 \ \mathrm{GeV}$

E_t/GeV
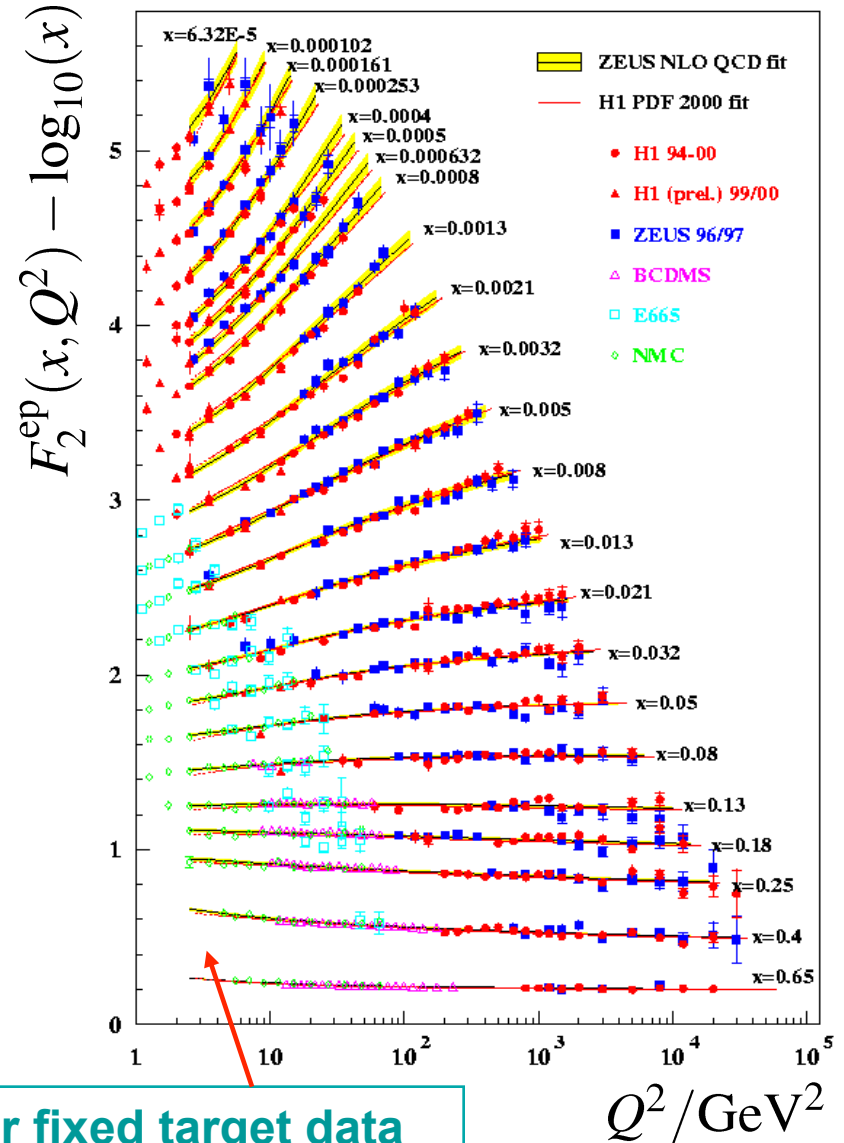
jet e^+

$F_2(x,Q^2)$ Results

★ **No evidence of rapid decrease of cross section at highest Q^2**

➡ $\boxed{R_{\text{quark}} < 10^{-18}\,\text{m}}$
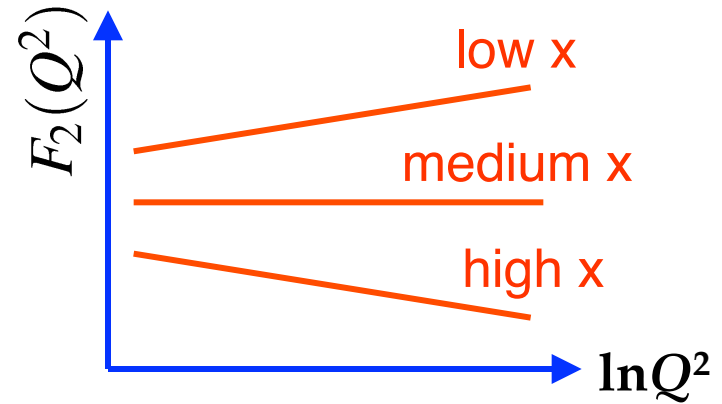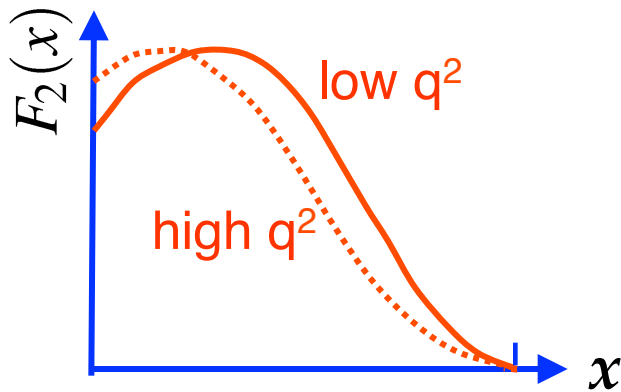
★ **For $x > 0.05$, only weak dependence of F_2 on Q^2 : consistent with the expectation from the quark-parton model**

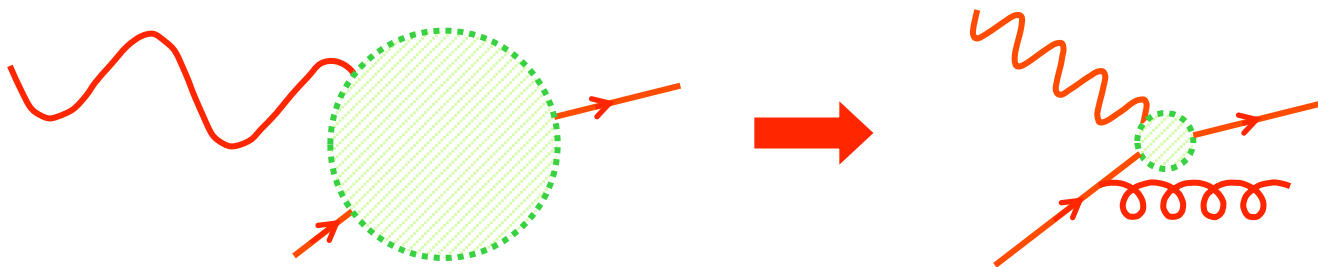★ **But observe clear scaling violations, particularly at low x**

$$F_2(x,Q^2) \neq F_2(x)$$

Earlier fixed target data

Origin of Scaling Violations

★ **Observe "small" deviations from exact Bjorken scaling** $\quad F_2(x) \to F_2(x, Q^2)$

★ **At high Q^2 observe more low x quarks**

★ **"Explanation": at high Q^2 (shorter wave-length) resolve finer structure: i.e. reveal quark is sharing momentum with gluons. At higher Q^2 expect to "see" more low x quarks**

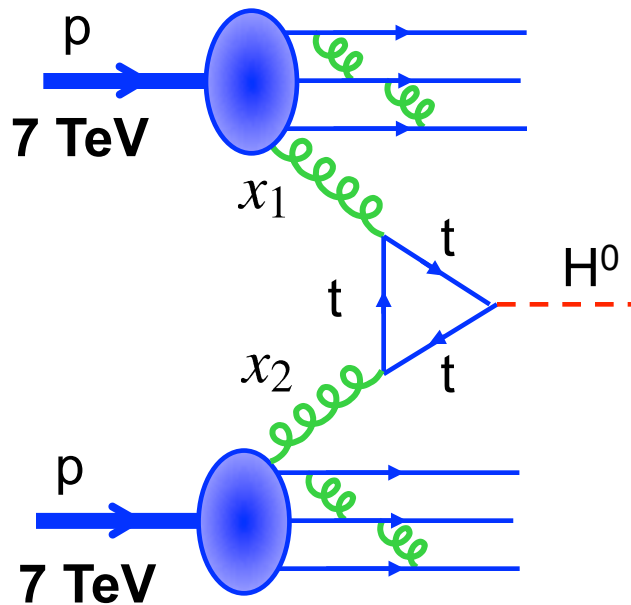★ **QCD cannot predict the x dependence of $F_2(x, Q^2)$**

★ **But QCD can predict the Q^2 dependence of $F_2(x, Q^2)$**

Proton-Proton Collisions at the LHC

★ **Measurements of structure functions not only provide a powerful test of QCD, the parton distribution functions are essential for the calculation of cross sections at** pp **and** p$\overline{\text{p}}$ **colliders.**

• **Example:** **Higgs production at the Large Hadron Collider LHC (2009-)**

  • **The LHC will collide 7 TeV protons on 7 TeV protons**

  • **However underlying collisions are between partons**

  • **Higgs production the LHC dominated by "gluon-gluon fusion"**

• **Cross section depends on gluon PDFs**

$$\sigma(pp \to HX) \sim \int_0^1 \int_0^1 g(x_1)g(x_2)\sigma(gg \to H)\mathrm{d}x_1\mathrm{d}x_2$$

• **Uncertainty in gluon PDFs lead to a ±5 % uncertainty in Higgs production cross section**

• **Prior to HERA data uncertainty was ±25 %**

Summary

♦ **At very high electron energies** $\lambda \ll r_p$:
  the proton appears to be a sea of
  quarks and gluons.

♦ **Deep Inelastic Scattering = Elastic scattering
  from the quasi-free constituent quarks**

  ➡ **Bjorken Scaling** $F_1(x, Q^2) \to F_1(x)$ | **point-like scattering** |

  ➡ **Callan-Gross** $F_2(x) = 2xF_1(x)$ | **Scattering from spin-1/2** |

♦ **Describe scattering in terms of parton distribution functions** $u(x), d(x), \ldots$
  which describe momentum distribution inside a nucleon

♦ **The proton is much more complex than just uud - sea of anti-quarks/gluons**

♦ **Quarks carry only 50 % of the protons momentum – the rest is due to
  low energy gluons**

♦ **We will come back to this topic when we discuss neutrino scattering…**