

#	Dates	Title	Exercise	(1=easy, 10=hard)	Tasks
1	22.2	Introduction	VESTA	2-3	Read Chap. 1
2	01.3	Crystal structures	Daniel - info	4	Read Chap. 2, Ex. 1
3	08.03	Reciprocal space	Discuss Ex. 1	6	Read Chap. 2, Ex. 2
4	15.03	Scattering Theory	Discuss Ex. 2	8-9	Read Chap. 3, Ex. 3
5	22.03	Crystal bindings	Discuss Ex. 3	5	Read Chap. 4, Ex. 4
6	29.03	Phonons	Discuss Ex. 4	5-6	Read Chap. 5, Ex. 5
7	05.04	Thermal properties	Discuss Ex. 5	5-6	Read Chap. 6, Ex. 6
8	12.04	Electron gasses, $C_{el}$	Discuss Ex. 6	5-6	
--	19.04	EASTER HOLIDAY	-----	0	<b>RECAP</b>
9	26.04	Electronic band struc.	Discuss Ex. 7	5-6	
10	03.05	Semi-conductors	Discuss Ex. 8	6	
11	10.05	Fermi surfaces & Metals - I	Discuss Ex. 9	8	
12	17.05	Fermi surfaces & Metals - II	Discuss Ex. 10	8	
13	24.05	Guest lecture	Discuss Ex. 11	--	
14	31.05	Repetition		4	

# Today's Lecture: Heat Capacity

1. Introduction: Why do we care?

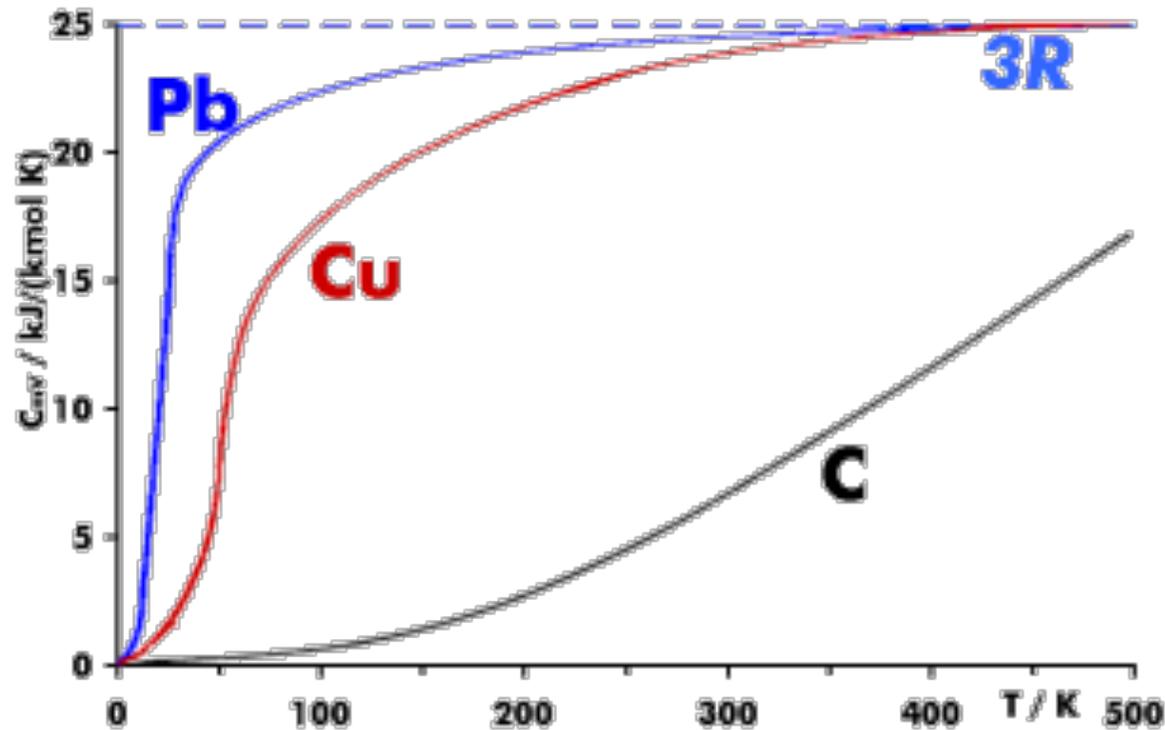
2. Debye's Model for heat capacity (specific heat).

3. Einstein's Model for heat capacity.

4. How to measure heat capacity.

5. Thermal conductivity.

# Heat Capacity: Petit Dulong's Law / Gesetz

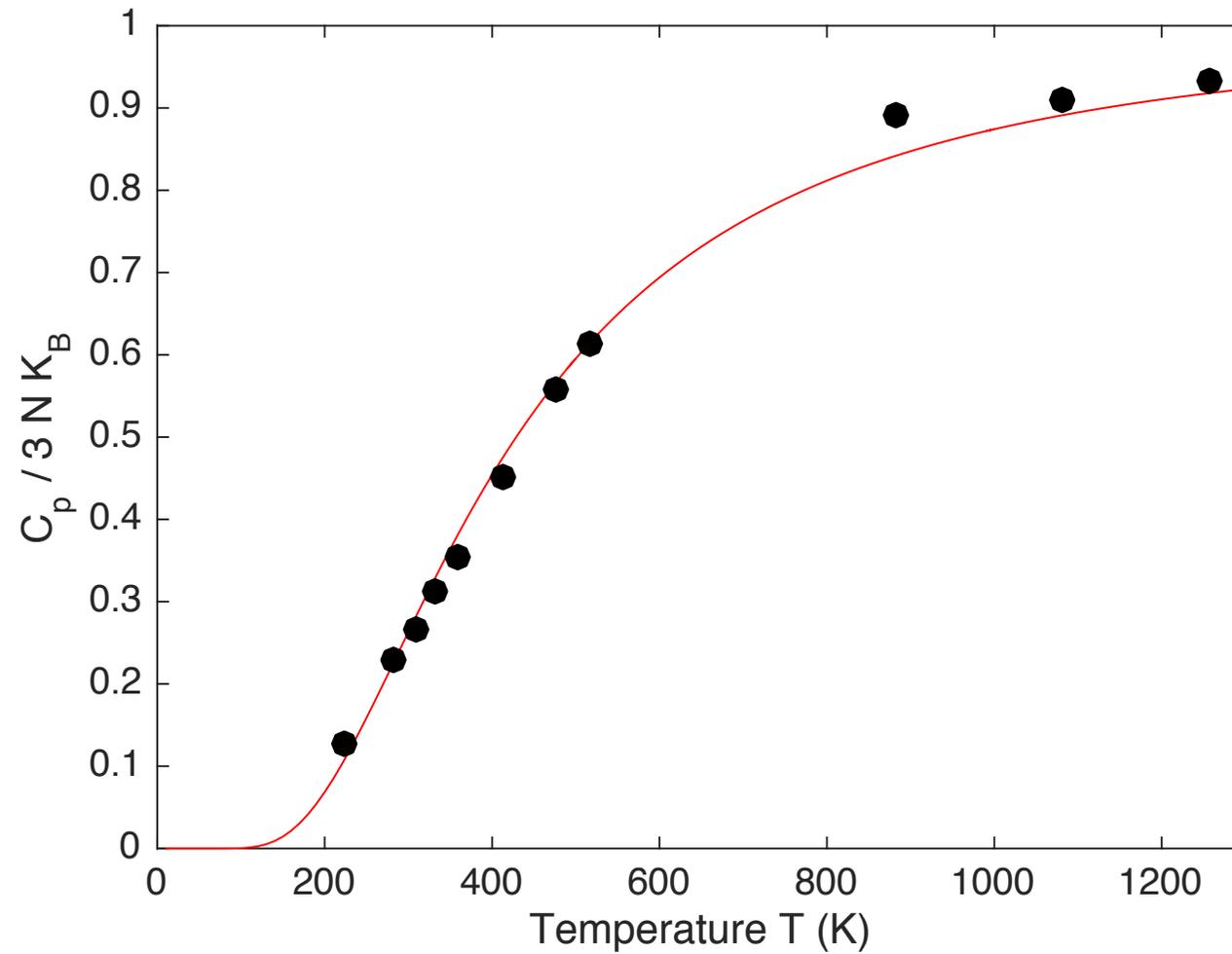


$$N = n N_a \text{ (# moles x Avogadro's Number)}$$

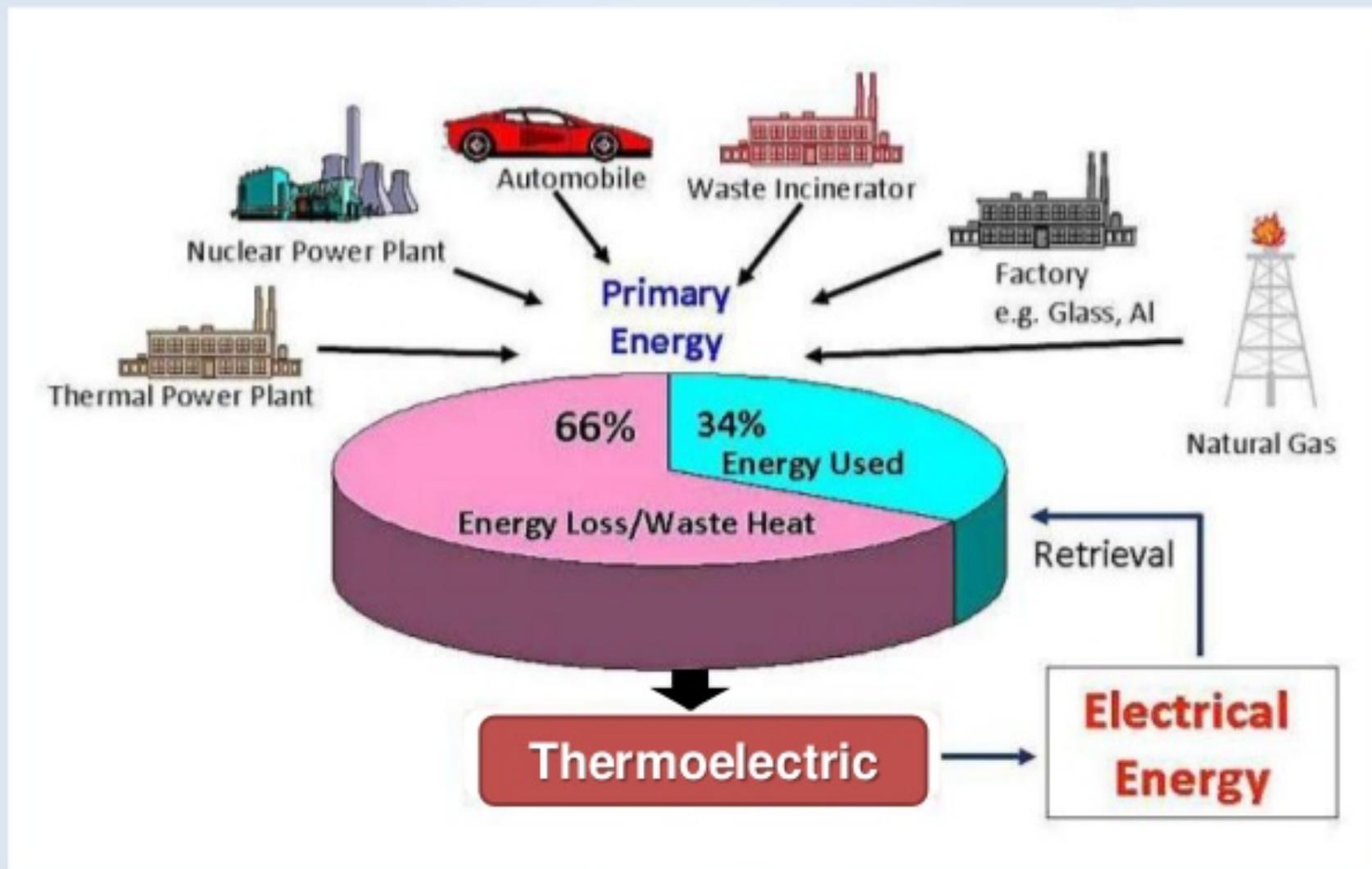
$$R = N_a k_B$$

$$C/n = 3R / n = 3 N_a k_B / n = 3Nk_B$$

# Heat Capacity: Diamond (as of 1906)



# Waste Heat to Electricity



# Today's Lecture: Heat Capacity

1. Introduction: Why do we care?

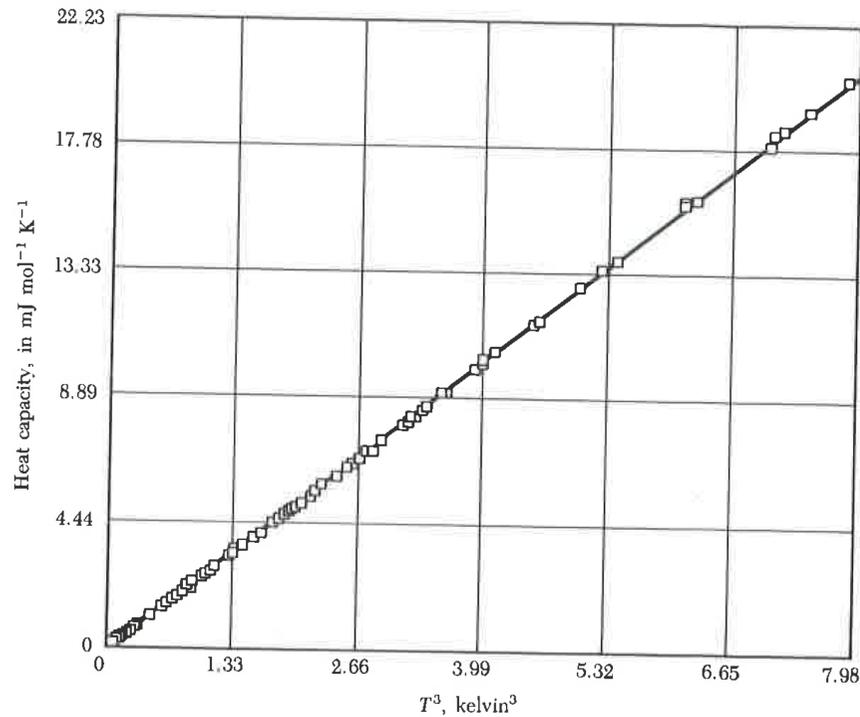
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# Heat Capacity: Argon



**Figure 9** Low temperature heat capacity of solid argon, plotted against  $T^3$ . In this temperature region the experimental results are in excellent agreement with the Debye  $T^3$  law with  $\theta = 92.0$  K. (Courtesy of L. Finegold and N. E. Phillips.)

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## Low-Temperature Heat Capacities of Solid Argon and Krypton\*

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University of California, Berkeley, California 94720*

(Received 3 August 1968)

# Debye Temperatures

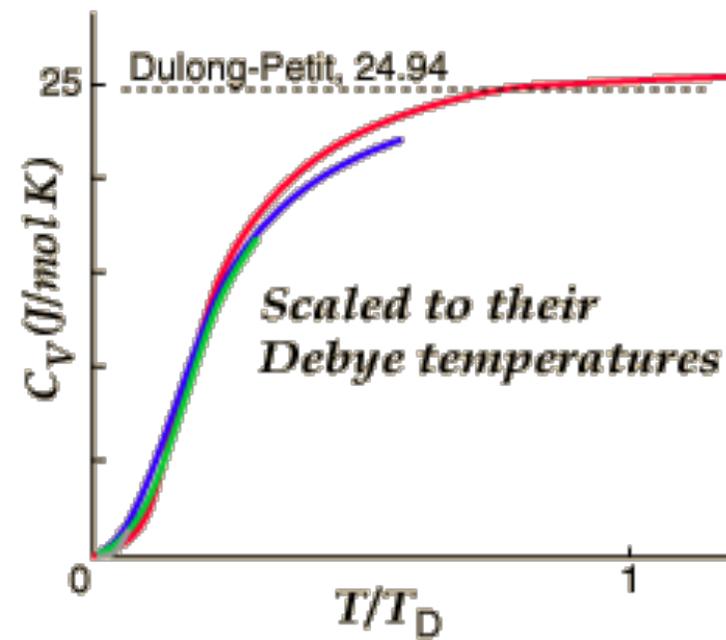
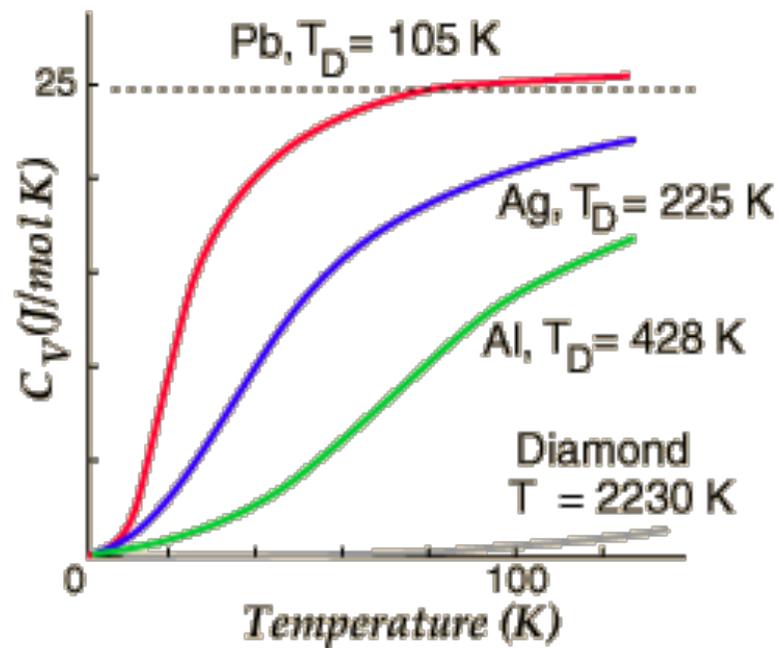
**TABLE 1 Debye Temperature and Thermal Conductivity\***

Low temperature limit of  $\theta$ , in Kelvin  
Thermal conductivity at 300 K, in  $\text{W cm}^{-1}\text{K}^{-1}$

Li 344 0.85	Be 1440 2.00												B 0.27	C 2230 1.29	N	O	F	Ne 75
Na 158 1.41	Mg 400 1.56												Al 428 2.37	Si 645 1.48	P	S	Cl	Ar 92
K 91 1.02	Ca 230	Sc 360 0.16	Ti 420 0.22	V 380 0.31	Cr 630 0.94	Mn 410 0.08	Fe 470 0.80	Co 445 1.00	Ni 450 0.91	Cu 343 4.01	Zn 327 1.16	Ga 320 0.41	Ge 374 0.60	As 282 0.50	Se 90 0.02	Br	Kr 72	
Rb 56 0.58	Sr 147	Y 280 0.17	Zr 291 0.23	Nb 275 0.54	Mo 450 1.38	Tc 0.51	Ru 600 1.17	Rh 480 1.50	Pd 274 0.72	Ag 225 4.29	Cd 209 0.97	In 108 0.82	Sn 200 0.67	Sb 211 0.24	Te 153 0.02	I	Xe 64	
Cs 38 0.36	Ba 110	La $\beta$ 142 0.14	Hf 252 0.23	Ta 240 0.58	W 400 1.74	Re 430 0.48	Os 500 0.88	Ir 420 1.47	Pt 240 0.72	Au 165 3.17	Hg 71.9	Tl 78.5 0.46	Pb 105 0.35	Bi 119	Po	At	Rn	
Fr	Ra	Ac	Ce 0.11	Pr 0.12	Nd 0.16	Pm	Sm 0.13	Eu	Gd 200 0.11	Tb 0.11	Dy 210 0.11	Ho 0.16	Er 0.14	Tm 0.17	Yb 120 0.35	Lu 210 0.16		
			Th 163 0.54	Pa	U 207 0.28	Np 0.06	Pu 0.07	Am	Cm	Bk	Cf	Es	Fm	Md	No	Lr		

\* Most of the  $\theta$  values were supplied by N. Pearlman; references are given the *A.I.P. Handbook*, 3rd ed, the thermal conductivity values are from R. W. Powell and Y. S. Touloukian, *Science* **181**, 999 (1973).

# Heat Capacity for different elements



# Exercise

## Exercise 3 *Singularity in density of states*

- a) From the dispersion relation derived in the lecture for a monoatomic linear lattice of  $N$  atoms with nearest neighbour interactions, show that the density of modes is

$$D(\omega) = \frac{2N}{\pi} \cdot \frac{1}{\sqrt{\omega_m^2 - \omega^2}}, \quad (4)$$

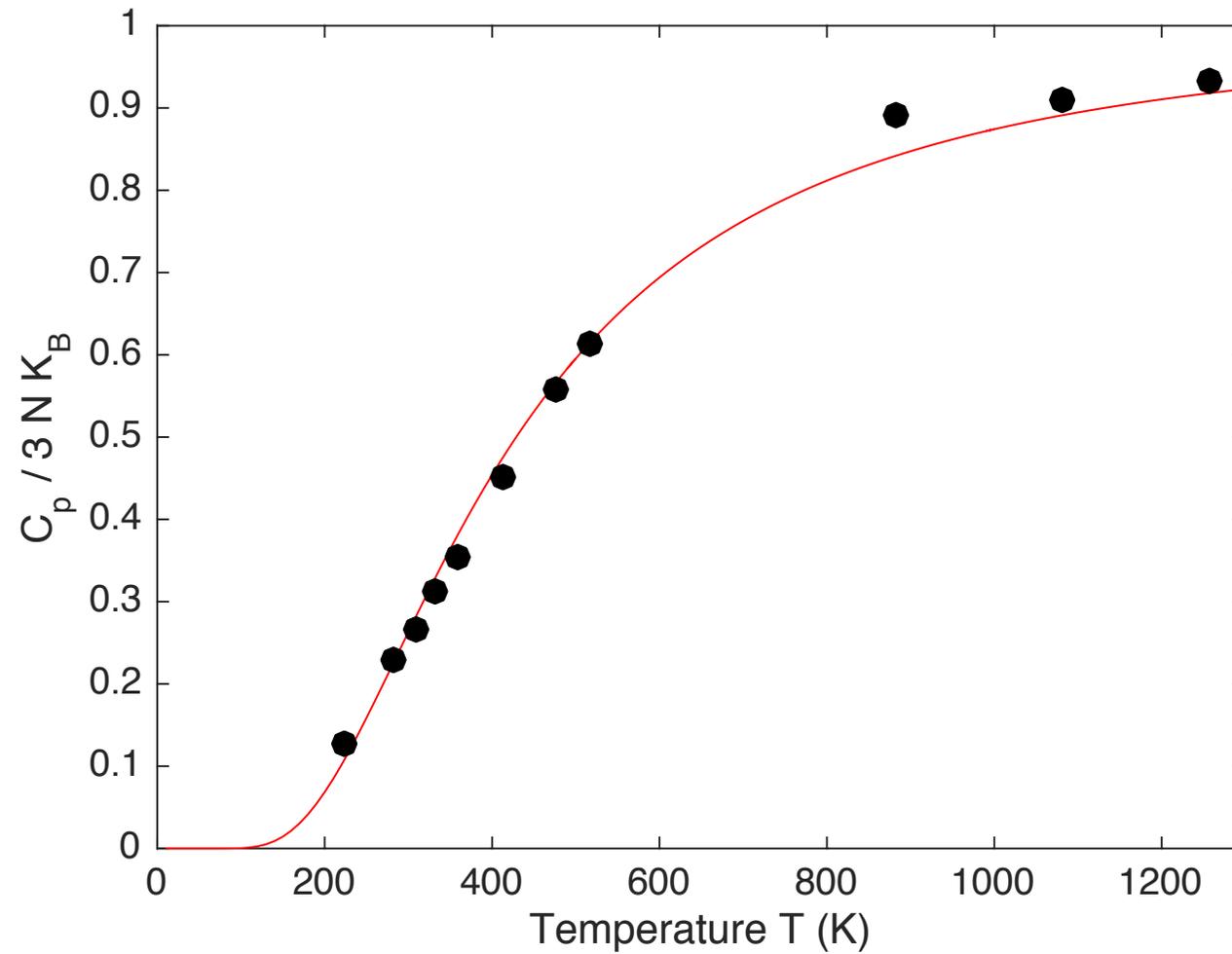
where  $\omega_m$  is the maximum frequency.

- b) Make a plot of equation (4).
- c) Suppose that an optical phonon branch has the form  $\omega(K) = \omega_0 - AK^2$ , near  $K = 0$  in three dimensions. Show that  $D(\omega) = \left(\frac{L}{2\pi}\right)^3 \left(\frac{2\pi}{A^{3/2}}\right) (\omega_0 - \omega)^{\frac{1}{2}}$  for  $\omega < \omega_0$  and  $D(\omega) = 0$  for  $\omega > \omega_0$ . Here the density of modes is discontinuous.

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# Heat Capacity: Diamond (as of 1906)



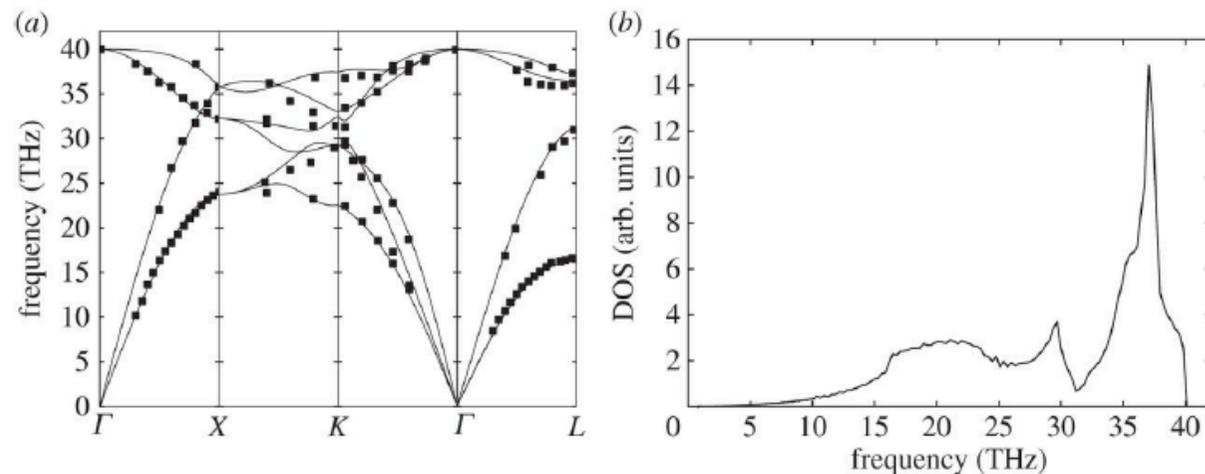
# Exercise

## Exercise 3 Heat capacity - Einstein model

Einstein derived the expression

$$C = 3Nk_B \left( \frac{\hbar\omega_0}{k_B T} \right)^2 \frac{\exp\left(\frac{\hbar\omega_0}{k_B T}\right)}{\left[ \exp\left(\frac{\hbar\omega_0}{k_B T}\right) - 1 \right]^2}. \quad (1)$$

- a) In figure 1, the heat capacity of diamond is plotted as  $\frac{C_p}{3Nk_B}$  versus temperature. Einstein's model has just one free parameter:  $\omega_0$ . Based on these data (given below), determine the energy scale  $\hbar\omega_0$  that would give the best description of the experiment.
- b) How does this energy scale compare with the energy scale of optical phonons in diamond (see figure 2)?



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# Heat Capacity: Experimental setup

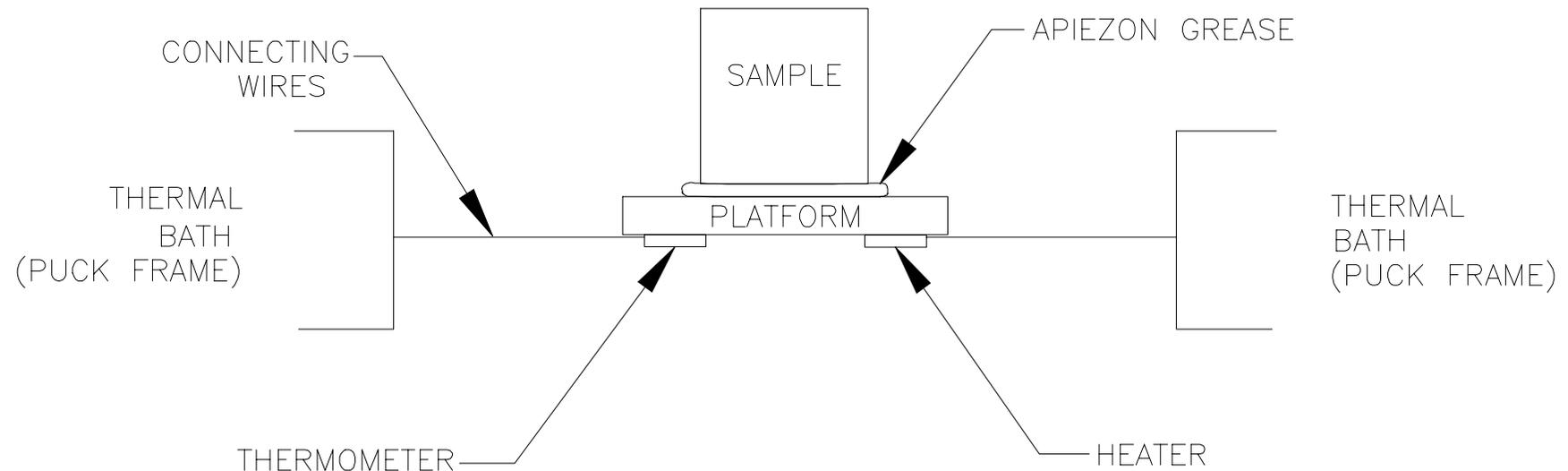


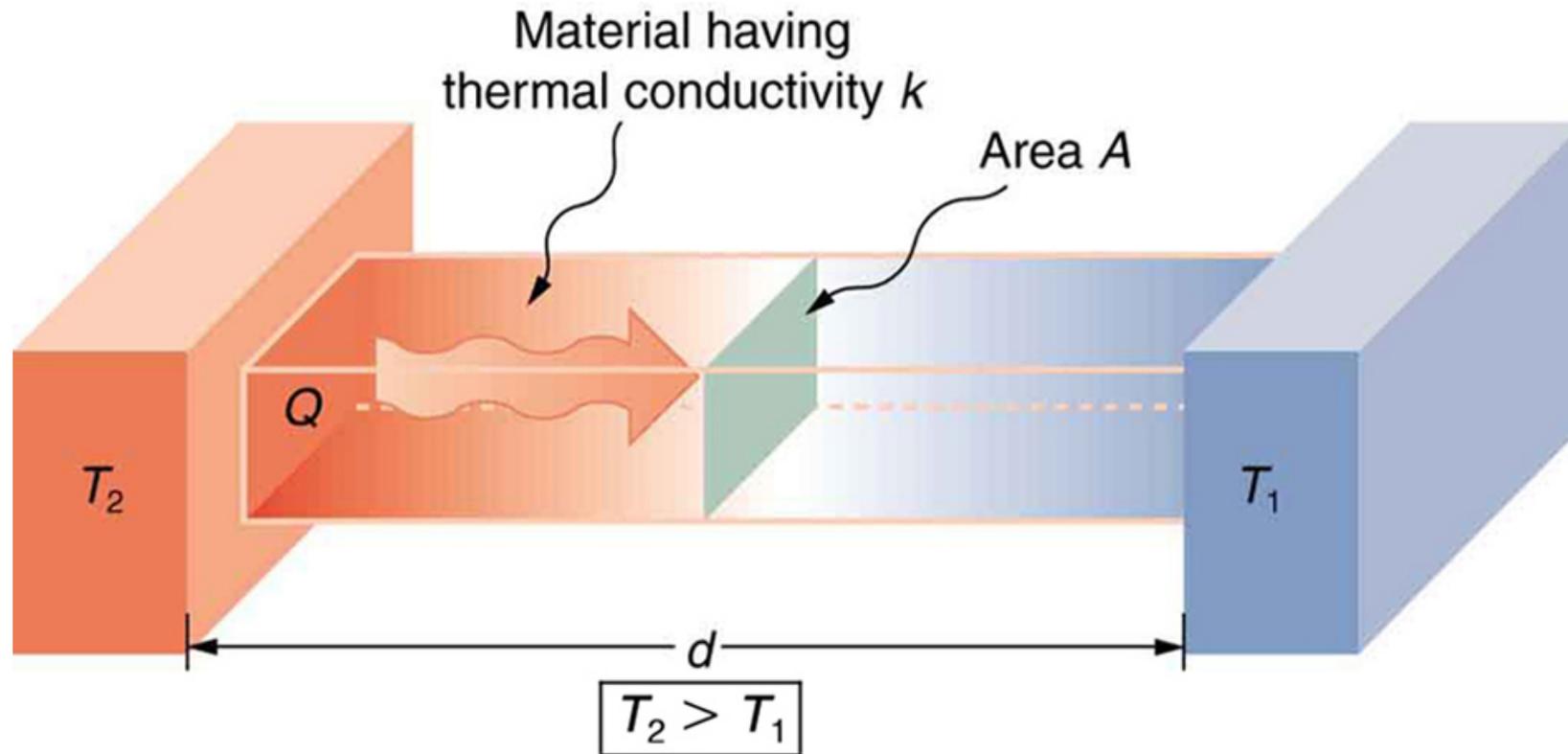
Figure 1-1. Thermal Connections to Sample and Sample Platform in PPMS Heat Capacity Option

**Prof. Andreas Schilling's Laboratory.  
Possibility for bachelor projects!**

# Today's Lecture: Heat Capacity

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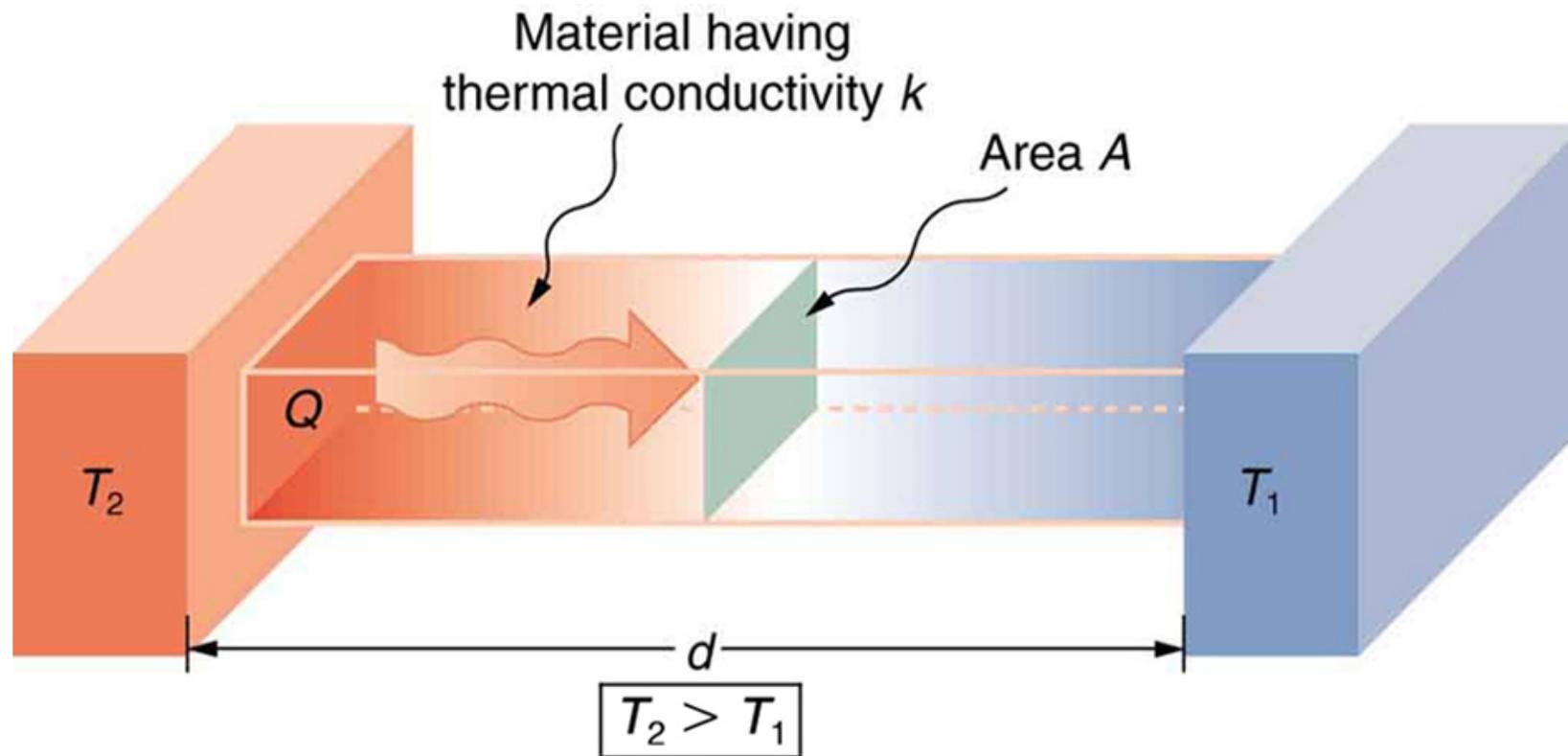
# Thermal conductivity $\kappa$



$$\kappa = \frac{d P}{A \Delta T} \quad \text{or} \quad \kappa \Delta T = \frac{d}{A} P$$

Where the power  $P=Q/t$  with  $t$  being time and  $Q = U$  the heat energy.

# $\kappa$ and $C_V$



$$\kappa = \frac{d P}{A \Delta T} \quad \text{or} \quad \kappa \Delta T = \frac{d}{A} P \quad \left. \begin{array}{l} \\ C \Delta T = \Delta U \end{array} \right\} \longrightarrow \kappa \propto C_V$$

# Thermal Conductivity: NaF

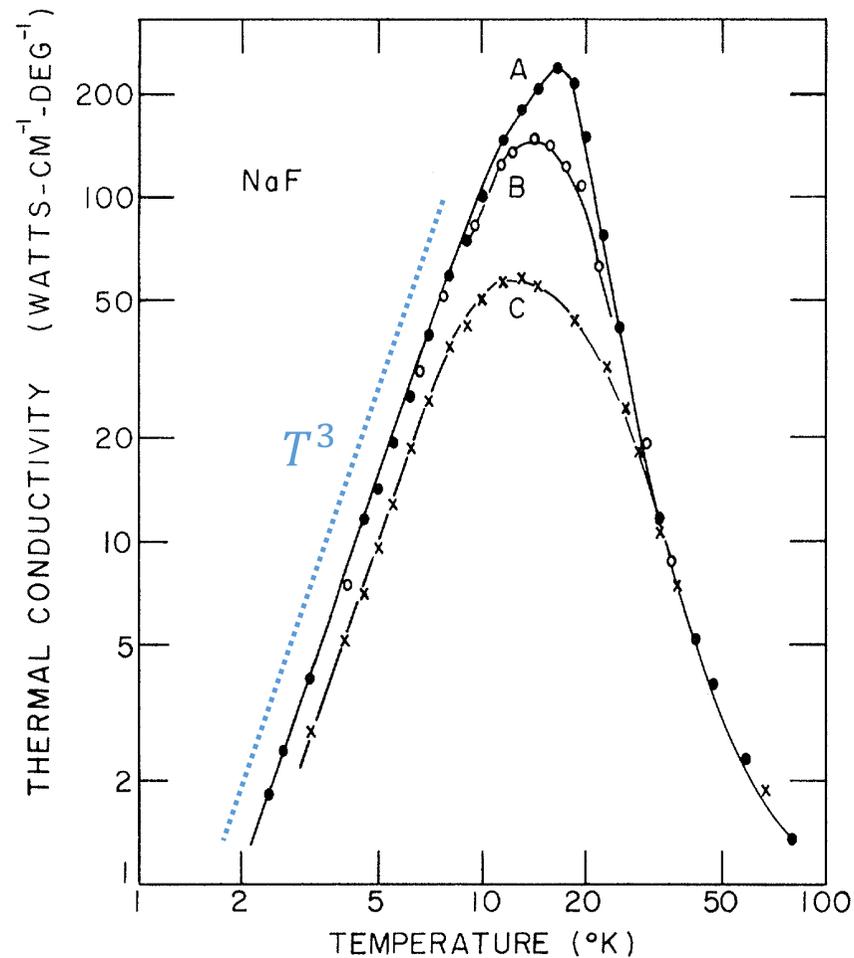
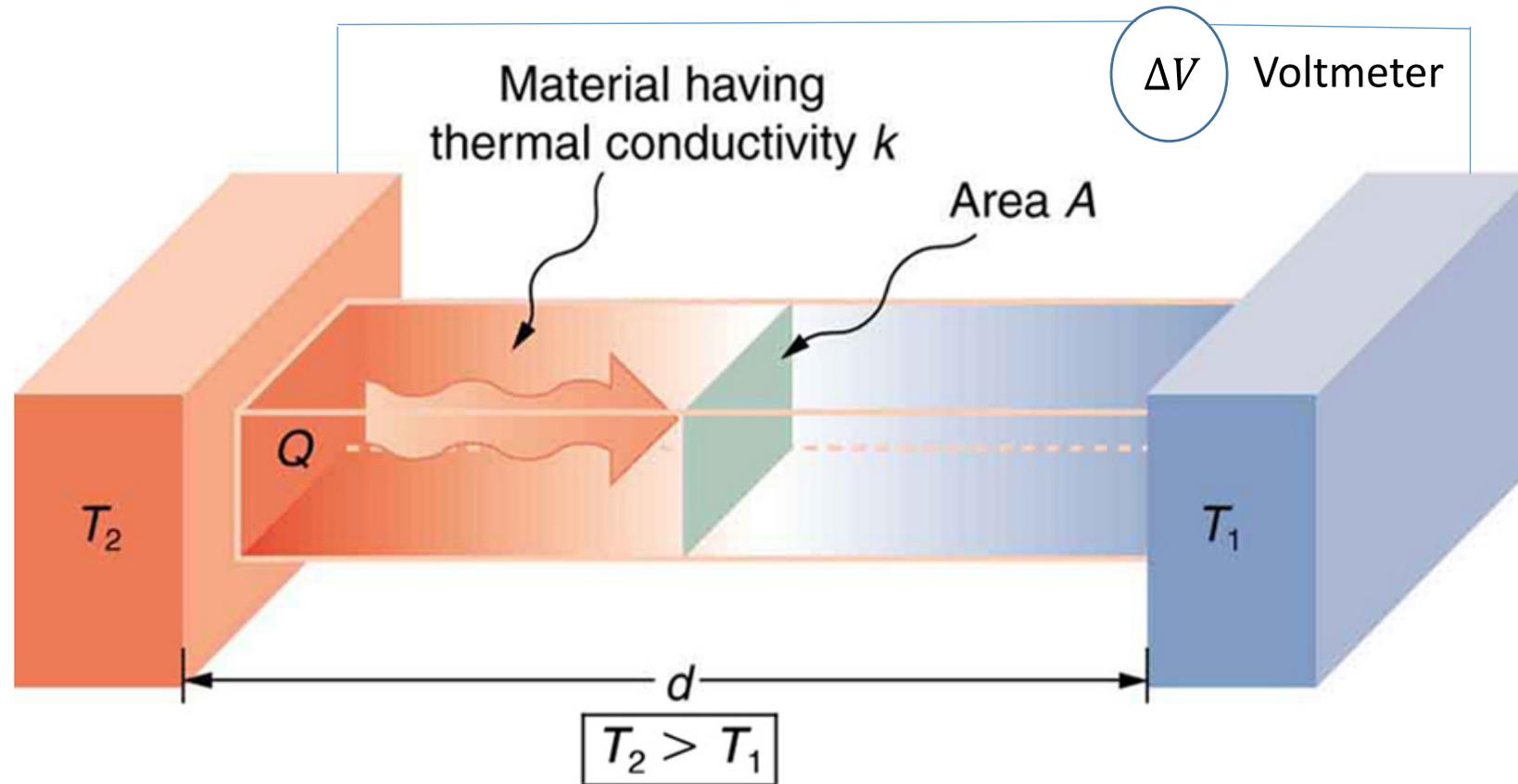


FIG. 1. Thermal conductivity versus temperature for pure NaF crystals. Curve A, NaF sample, this paper; curve B, NaF sample, Ref. 1; curve C, typical singly grown NaF (smaller cross section).

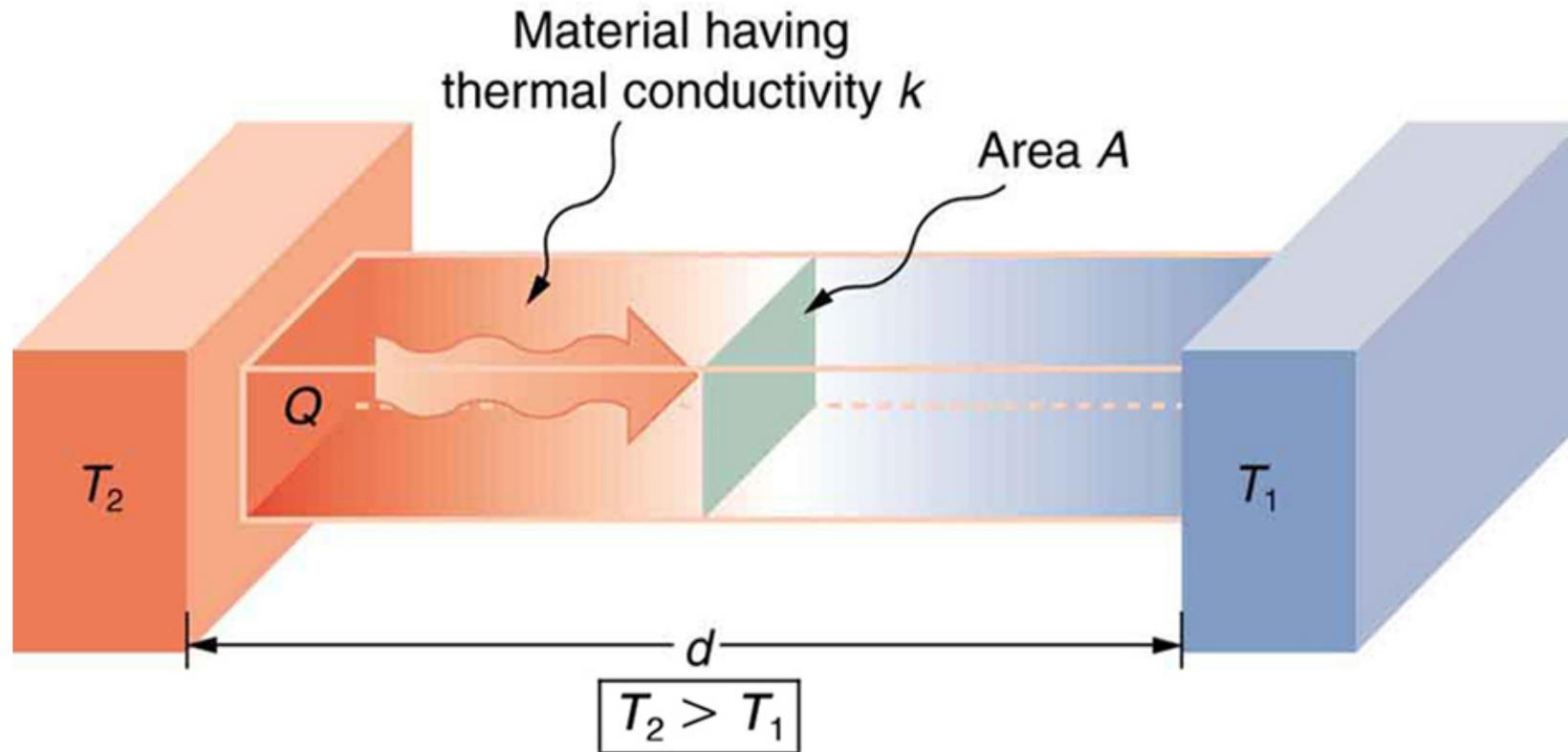
# Thermal conductivity $\kappa$



$$S = \frac{\Delta V}{\Delta T} \quad \text{or} \quad S \Delta T = \Delta V$$

Where the  $\Delta V$  is the thermoelectric voltage

# *S versus $\kappa$*



$$\kappa = \frac{d P}{A \Delta T} \quad \text{or} \quad \kappa \Delta T = \frac{d}{A} P$$

$$S = \frac{\Delta V}{\Delta T} \quad \text{or} \quad S \Delta T = \Delta V$$