Kern- und Teilchenphysik II
Lecture 4: Neutral Weak Interaction

(adapted from the Handout of Prof. Mark Thomson)

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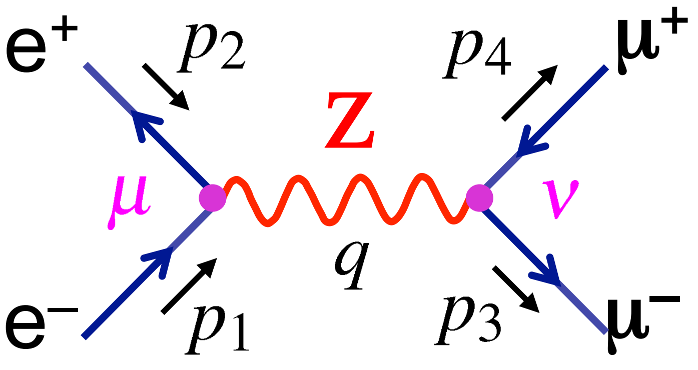
http://www.physik.uzh.ch/de/lehre/PHY213/FS2018.html

The Z resonance

- ★ **Want to calculate the cross-section for** $e^+e^- \rightarrow Z \rightarrow \mu^+\mu^-$
  - •**Feynman rules for the diagram below give:**



e⁺e⁻ vertex: $\overline{v}(p_2)\cdot -ig_Z\gamma^\mu\tfrac{1}{2}(c_V^e - c_A^e\gamma^5)\cdot u(p_1)$

Z propagator: $\dfrac{-ig_{\mu\nu}}{q^2-m_Z^2}$

μ⁺μ⁻ vertex: $\overline{u}(p_3)\cdot -ig_Z\gamma^\nu\tfrac{1}{2}(c_V^\mu - c_A^\mu\gamma^5)\cdot v(p_4)$

➡ $$-iM_{fi} = [\overline{v}(p_2)\cdot -ig_Z\gamma^\mu\tfrac{1}{2}(c_V^e - c_A^e\gamma^5)\cdot u(p_1)].\frac{-ig_{\mu\nu}}{q^2-m_Z^2}.[\overline{u}(p_3)\cdot -ig_Z\gamma^\nu\tfrac{1}{2}(c_V^\mu - c_A^\mu\gamma^5)\cdot v(p_4)]$$

➡ $$\boxed{M_{fi} = -\frac{g_Z^2}{q^2-m_Z^2}g_{\mu\nu}[\overline{v}(p_2)\gamma^\mu\tfrac{1}{2}(c_V^e - c_A^e\gamma^5)\cdot u(p_1)].[\overline{u}(p_3)\gamma^\nu\tfrac{1}{2}(c_V^\mu - c_A^\mu\gamma^5)\cdot v(p_4)]}$$

- ★ **Convenient to work in terms of helicity states by explicitly using the Z coupling to LH and RH chiral states (ultra-relativistic limit so helicity = chirality)**

$$\tfrac{1}{2}(c_V - c_A\gamma^5) = c_L\tfrac{1}{2}(1-\gamma^5) + c_R\tfrac{1}{2}(1+\gamma^5)$$

LH and RH projections operators

The Z resonance

hence $c_V = (c_L + c_R),\;\; c_A = (c_L - c_R)$

and
$$\begin{aligned}\tfrac{1}{2}(c_V - c_A\gamma^5) &= \tfrac{1}{2}(c_L + c_R - (c_L - c_R)\gamma^5)\\ &= c_L\tfrac{1}{2}(1-\gamma^5) + c_R\tfrac{1}{2}(1+\gamma^5)\end{aligned}$$

with $c_L = \tfrac{1}{2}(c_V + c_A),\;\; c_R = \tfrac{1}{2}(c_V - c_A)$

★ Rewriting the matrix element in terms of LH and RH couplings:

$$\begin{aligned}M_{fi} = -\frac{g_Z^2}{q^2-m_Z^2}g_{\mu\nu}&[c_L^e\bar{v}(p_2)\gamma^\mu\tfrac{1}{2}(1-\gamma^5)u(p_1) + c_R^e\bar{v}(p_2)\gamma^\mu\tfrac{1}{2}(1+\gamma^5)u(p_1)]\\ &\times[c_L^\mu\bar{u}(p_3)\gamma^\nu\tfrac{1}{2}(1-\gamma^5)v(p_4) + c_R^\mu\bar{u}(p_3)\gamma^\nu\tfrac{1}{2}(1+\gamma^5)v(p_4)]\end{aligned}$$

★ Apply projection operators remembering that in the ultra-relativistic limit

$$\tfrac{1}{2}(1-\gamma^5)u = u_\downarrow;\quad \tfrac{1}{2}(1+\gamma^5)u = u_\uparrow,\quad \tfrac{1}{2}(1-\gamma^5)v = v_\uparrow,\quad \tfrac{1}{2}(1+\gamma^5)v = v_\downarrow$$

$$\Rightarrow\quad \begin{aligned}M_{fi} = -\frac{g_Z}{q^2-m_Z^2}g_{\mu\nu}&[c_L^e\bar{v}(p_2)\gamma^\mu u_\downarrow(p_1) + c_R^e\bar{v}(p_2)\gamma^\mu u_\uparrow(p_1)]\\ &\times[c_L^\mu\bar{u}(p_3)\gamma^\nu v_\uparrow(p_4) + c_R^\mu\bar{u}(p_3)\gamma^\nu v_\downarrow(p_4)]\end{aligned}$$

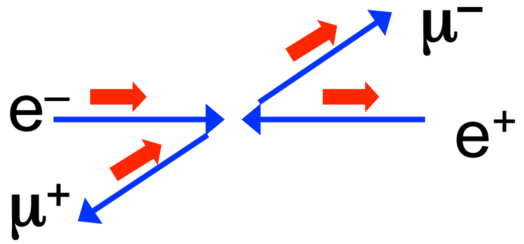
★ For a combination of **V** and **A** currents, $\bar{u}_\uparrow\gamma^\mu v_\uparrow = 0$ etc, gives four orthogonal contributions

$$\Rightarrow\quad \boxed{\begin{aligned}-\frac{g_Z^2}{q^2-m_Z^2}g_{\mu\nu}&[c_L^e\bar{v}_\uparrow(p_2)\gamma^\mu u_\downarrow(p_1) + c_R^e\bar{v}_\downarrow(p_2)\gamma^\mu u_\uparrow(p_1)]\\ &\times[c_L^\mu\bar{u}_\downarrow(p_3)\gamma^\nu v_\uparrow(p_4) + c_R^\mu\bar{u}_\uparrow(p_3)\gamma^\nu v_\downarrow(p_4)]\end{aligned}}$$

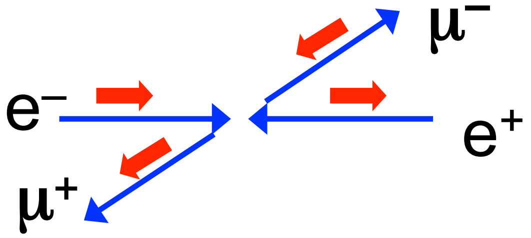
The Z resonance

- **Sum of 4 terms**

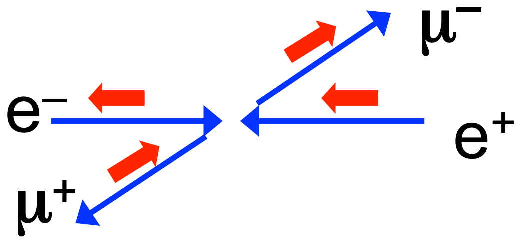
$$M_{RR} = -\frac{g_Z^2}{q^2-m_Z^2} c_R^e c_R^\mu g_{\mu\nu} [\overline{v}_\downarrow(p_2)\gamma^\mu u_\uparrow(p_1)][\overline{u}_\uparrow(p_3)\gamma^\nu v_\downarrow(p_4)]$$



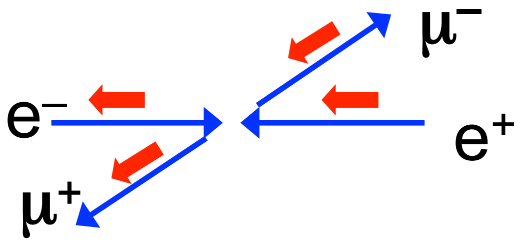
$$M_{RL} = -\frac{g_Z^2}{q^2-m_Z^2} c_R^e c_L^\mu g_{\mu\nu} [\overline{v}_\downarrow(p_2)\gamma^\mu u_\uparrow(p_1)][\overline{u}_\downarrow(p_3)\gamma^\nu v_\uparrow(p_4)]$$



$$M_{LR} = -\frac{g_Z^2}{q^2-m_Z^2} c_L^e c_R^\mu g_{\mu\nu} [\overline{v}_\uparrow(p_2)\gamma^\mu u_\downarrow(p_1)][\overline{u}_\uparrow(p_3)\gamma^\nu v_\downarrow(p_4)]$$



$$M_{LL} = -\frac{g_Z^2}{q^2-m_Z^2} c_L^e c_L^\mu g_{\mu\nu} [\overline{v}_\uparrow(p_2)\gamma^\mu u_\downarrow(p_1)][\overline{u}_\downarrow(p_3)\gamma^\nu v_\uparrow(p_4)]$$



Remember: the L/R refer to the helicities of the initial/final state particles

- **Fortunately we have calculated these terms before when considering** $e^+e^- \rightarrow \gamma \rightarrow \mu^+\mu^-$ **giving:**

$$[\overline{v}_\downarrow(p_2)\gamma^\mu u_\uparrow(p_1)][\overline{u}_\uparrow(p_3)\gamma^\nu v_\downarrow(p_4)] = s(1+\cos\theta) \quad \text{etc.}$$

The Z resonance

- **Applying the QED results to the Z exchange with gives:**

$$\frac{e^2}{q^2} \to \frac{g_Z^2}{q^2 - m_Z^2} c^e c^\mu$$

where $q^2 = s = 4E_e^2$

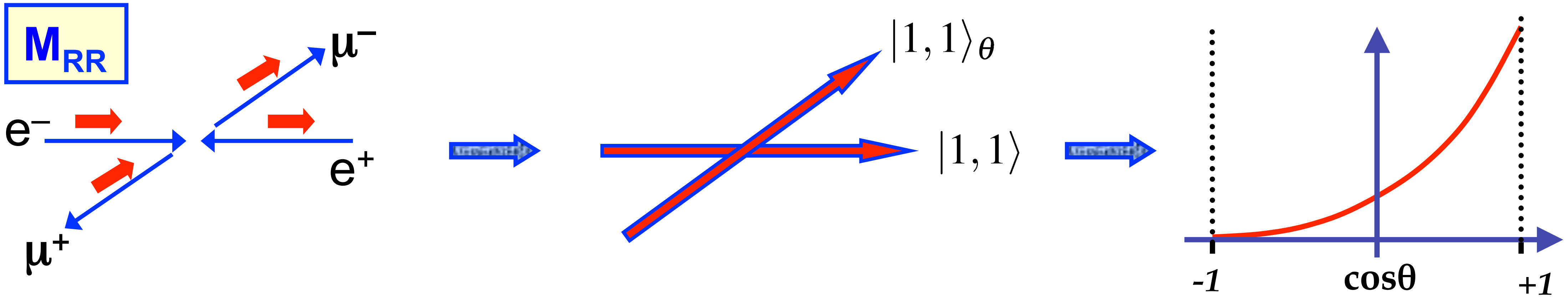
$$|M_{RR}|^2 = s^2 \left| \frac{g_Z^2}{s - m_Z^2} \right|^2 (c_R^e)^2 (c_R^\mu)^2 (1 + \cos\theta)^2$$

$$|M_{RL}|^2 = s^2 \left| \frac{g_Z^2}{s - m_Z^2} \right|^2 (c_R^e)^2 (c_L^\mu)^2 (1 - \cos\theta)^2$$

$$|M_{LR}|^2 = s^2 \left| \frac{g_Z^2}{s - m_Z^2} \right|^2 (c_L^e)^2 (c_R^\mu)^2 (1 - \cos\theta)^2$$

$$|M_{LR}|^2 = s^2 \left| \frac{g_Z^2}{s - m_Z^2} \right|^2 (c_L^e)^2 (c_L^\mu)^2 (1 + \cos\theta)^2$$

- **As before, the angular dependence of the matrix elements can be understood in terms of the spins of the incoming and outgoing particles e.g.**



The Breit-Wigner resonance

- ★ Need to consider carefully the propagator term $1/(s-m_Z^2)$ which diverges when the C.o.M. energy is equal to the rest mass of the Z boson
- ★ To do this need to account for the fact that the Z boson is an unstable particle
  - •For a stable particle at rest the time development of the wave-function is:

$$\psi \sim e^{-imt}$$

  - •For an unstable particle this must be modified to

$$\psi \sim e^{-imt} e^{-\Gamma t/2}$$

so that the particle probability decays away exponentially

$$\psi^* \psi \sim e^{-\Gamma t} = e^{-t/\tau} \qquad \text{with} \qquad \tau = \frac{1}{\Gamma_Z}$$

  - •Equivalent to making the replacement

$$m \rightarrow m - i\Gamma/2$$

- ★In the Z boson propagator make the substitution:

$$m_Z \rightarrow m_Z - i\Gamma_Z/2$$

- ★ Which gives:

$$(s-m_Z^2) \longrightarrow [s-(m_Z - i\Gamma_Z/2)] = s - m_Z^2 + im_Z\Gamma_Z + \tfrac{1}{4}\Gamma_Z^2 \approx s - m_Z^2 + im_Z\Gamma_Z$$

where it has been assumed that $\Gamma_Z \ll m_Z$

- ★ Which gives

$$\left|\frac{1}{s-m_Z^2}\right|^2 \rightarrow \left|\frac{1}{s-m_Z^2+im_Z\Gamma_Z}\right|^2 = \frac{1}{(s-m_Z^2)^2 + m_Z^2\Gamma_Z^2}$$

The Breit-Wigner resonance

- ★ **And the Matrix elements become**

$$|M_{RR}|^2 = \frac{g_Z^4 s^2}{(s-m_Z^2)^2 + m_Z^2\Gamma_Z^2}(c_R^e)^2(c_R^\mu)^2(1+\cos\theta)^2$$

etc.

- ★ **In the limit where initial and final state particle mass can be neglected:**

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \frac{1}{64\pi^2 s}|M_{fi}|^2$$

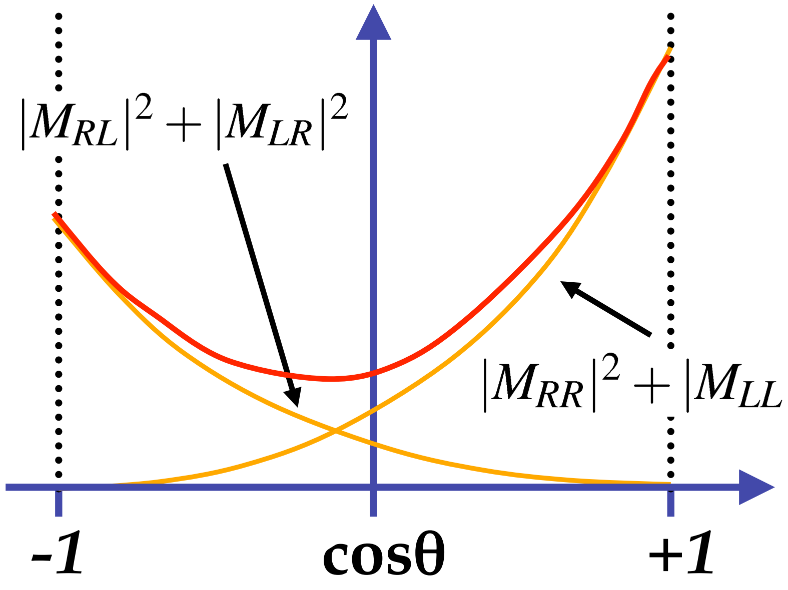
- ★ **Giving:**

$$\frac{\mathrm{d}\sigma_{RR}}{\mathrm{d}\Omega} = \frac{1}{64\pi^2}\frac{g_Z^4 s}{(s-m_Z^2)^2 + m_Z^2\Gamma_Z^2}(c_R^e)^2(c_R^\mu)^2(1+\cos\theta)^2$$

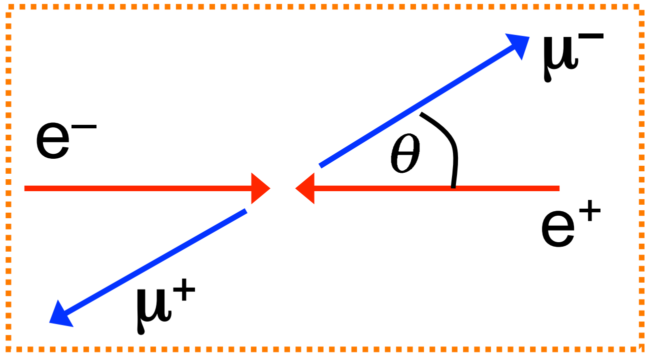
$$\frac{\mathrm{d}\sigma_{LL}}{\mathrm{d}\Omega} = \frac{1}{64\pi^2}\frac{g_Z^4 s}{(s-m_Z^2)^2 + m_Z^2\Gamma_Z^2}(c_L^e)^2(c_L^\mu)^2(1+\cos\theta)^2$$

$$\frac{\mathrm{d}\sigma_{LR}}{\mathrm{d}\Omega} = \frac{1}{64\pi^2}\frac{g_Z^4 s}{(s-m_Z^2)^2 + m_Z^2\Gamma_Z^2}(c_L^e)^2(c_R^\mu)^2(1-\cos\theta)^2$$

$$\frac{\mathrm{d}\sigma_{RL}}{\mathrm{d}\Omega} = \frac{1}{64\pi^2}\frac{g_Z^4 s}{(s-m_Z^2)^2 + m_Z^2\Gamma_Z^2}(c_L^e)^2(c_R^\mu)^2(1-\cos\theta)^2$$



- ★ **Because $|M_{LL}|^2 + |M_{RR}|^2 \neq |M_{LR}|^2 + |M_{RL}|^2$, the differential cross section is asymmetric, i.e. parity violation (although not maximal as was the case for the W boson).**



Cross-section

- ★ To calculate the total cross section need to sum over all matrix elements and **average** over the **initial spin states**. Here, assuming unpolarized beams (i.e. both **e⁺** and both **e⁻** spin states equally likely) there a four combinations of initial electron/positron spins, so

$$
\begin{aligned}
\langle |M_{fi}|^2\rangle &= \frac{1}{2}.\frac{1}{2}.(|M_{RR}|^2+|M_{LL}|^2+|M_{LR}|^2+|M_{RL}|^2) \\
&= \tfrac{1}{2}.\tfrac{1}{2}\frac{g_Z^4 s^2}{(s-m_Z^2)^2+m_Z^2\Gamma_Z^2}\times\{[(c_R^e)^2(c_R^\mu)^2+(c_L^e)^2(c_L^2)^2](1+\cos\theta)^2 \\
&\qquad +[(c_L^e)^2(c_R^\mu)^2+(c_R^e)^2(c_L^2)^2](1-\cos\theta)^2\}
\end{aligned}
$$

- ★ The part of the expression {…} can be rearranged:

$$
\begin{aligned}
\{\ldots\} = {} & [(c_R^e)^2+(c_L^e)^2][(c_R^\mu)^2+(c_L^\mu)^2](1+\cos^2\theta) \\
& +2[(c_R^e)^2-(c_L^e)^2][(c_R^\mu)^2-(c_L^\mu)^2]\cos\theta
\end{aligned}
$$

and using $c_V^2+c_A^2=2(c_L^2+c_R^2)$ and $c_V c_A = c_L^2+c_R^2$

$$
\{\ldots\} = \frac{1}{4}[(c_V^e)^2+(c_A^e)^2][(c_V^\mu)^2+(c_A^\mu)^2](1+\cos^2\theta)+2c_V^e c_A^e c_V^\mu c_A^\mu\cos\theta
$$

Cross-section

★**Hence the complete expression for the unpolarized differential cross section is:**

$$\begin{aligned}\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} &= \frac{1}{64\pi^2 s}\langle|M_{fi}|^2\rangle \\ &= \frac{1}{64\pi^2}\cdot\frac{1}{4}\cdot\frac{g_Z^4 s}{(s-m_Z^2)^2+m_Z^2\Gamma_Z^2}\times \\ &\quad \left\{\tfrac{1}{4}[(c_V^e)^2+(c_A^e)^2][(c_V^\mu)^2+(c_A^\mu)^2](1+\cos^2\theta)+2c_V^e c_A^e c_V^\mu c_A^\mu \cos\theta\right\}\end{aligned}$$

★ **Integrating over solid angle** $\mathrm{d}\Omega = \mathrm{d}\phi\,\mathrm{d}(\cos\theta) = 2\pi\mathrm{d}(\cos\theta)$

$\int_{-1}^{+1}(1+\cos^2\theta)\mathrm{d}(\cos\theta) = \int_{-1}^{+1}(1+x^2)dx = \frac{8}{3}$ **and** $\int_{-1}^{+1}\cos\theta\,\mathrm{d}(\cos\theta) = 0$

$$\boxed{\sigma_{e^+e^-\to Z\to\mu^+\mu^-} = \frac{1}{192\pi}\frac{g_Z^4 s}{(s-m_Z^2)^2+m_Z^2\Gamma_Z^2}\left[(c_V^e)^2+(c_A^e)^2\right]\left[(c_V^\mu)^2+(c_A^\mu)^2\right]}$$

★ **Note: the total cross section is proportional to the sums of the squares of the vector- and axial-vector couplings of the initial and final state fermions**

$$(c_V^f)^2+(c_A^f)^2$$

Cross section

★ **Can write the total cross section**

$$\boxed{\sigma_{e^+e^-\to Z\to\mu^+\mu^-}=\frac{1}{192\pi}\frac{g_Z^4 s}{(s-m_Z^2)^2+m_Z^2\Gamma_Z^2}\left[(c_V^e)^2+(c_A^e)^2\right]\left[(c_V^\mu)^2+(c_A^\mu)^2\right]}$$

in terms of the Z boson decay rates (partial widths)

$$\Gamma(Z\to e^+e^-)=\frac{g_Z^2 m_Z}{48\pi}\left[(c_V^e)^2+(c_A^e)^2\right]\quad\textbf{and}\quad\Gamma(Z\to\mu^+\mu^-)=\frac{g_Z^2 m_Z}{48\pi}\left[(c_V^\mu)^2+(c_A^\mu)^2\right]$$

⇨ $$\sigma=\frac{12\pi}{m_Z^2}\frac{s}{(s-m_Z^2)^2+m_Z^2\Gamma_Z^2}\Gamma(Z\to e^+e^-)\Gamma(Z\to\mu^+\mu^-)$$

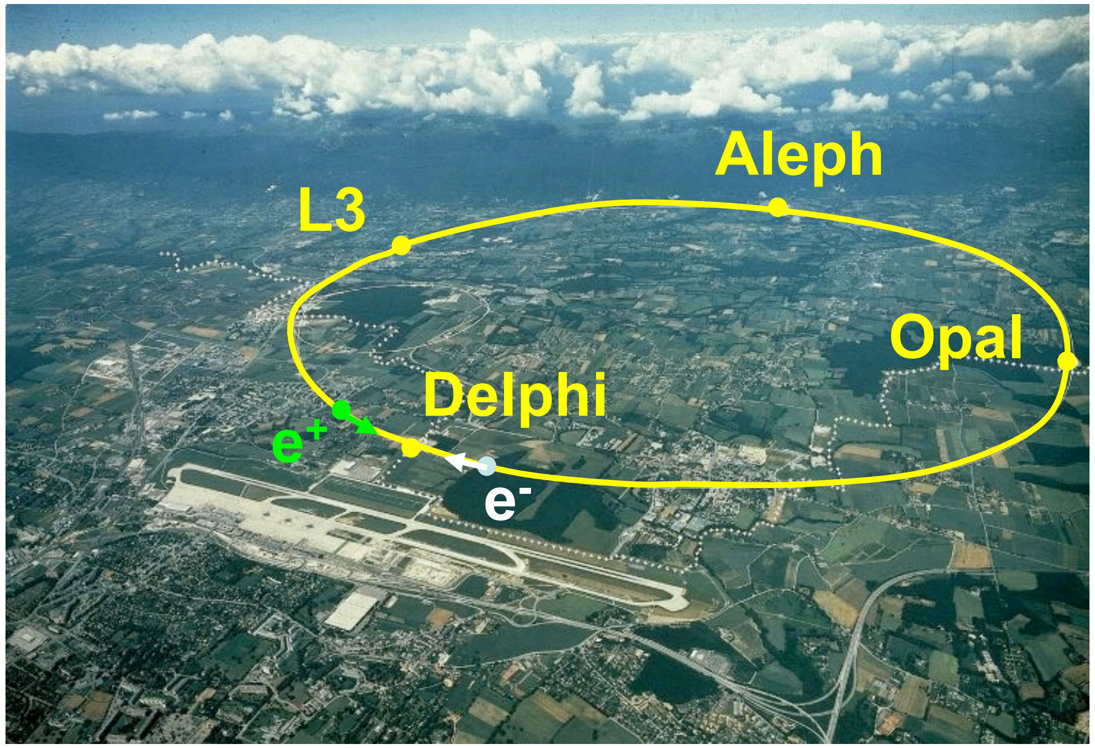
★ **Writing the partial widths as $\Gamma_{ee}=\Gamma(Z\to e^+e^-)$ etc., the total cross section can be written**

$$\boxed{\sigma(e^+e^-\to Z\to f\overline{f})=\frac{12\pi}{m_Z^2}\frac{s}{(s-m_Z^2)^2+m_Z^2\Gamma_Z^2}\Gamma_{ee}\Gamma_{ff}}$$

where f is the final state fermion flavour

EW Measurements at LEP

★The **Large Electron Positron (LEP) Collider at CERN (1989-2000) was designed to make precise measurements of the properties of the Z and W bosons.**



- **26 km circumference accelerator straddling French/Swiss boarder**
- **Electrons and positrons collided at 4 interaction points**
- **4 large detector collaborations (each with 300-400 physicists):**
  - **ALEPH,**
  - **DELPHI,**
  - **L3,**
  - **OPAL**

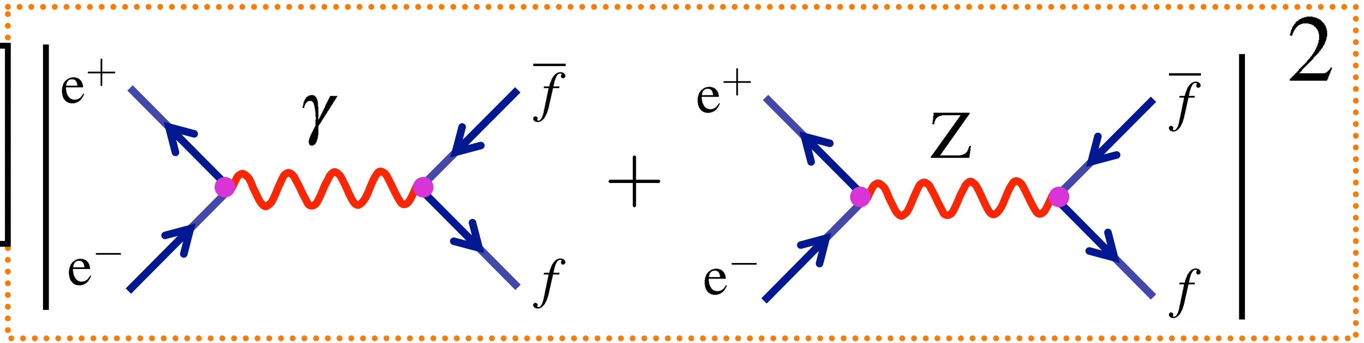
Basically a large Z and W factory:

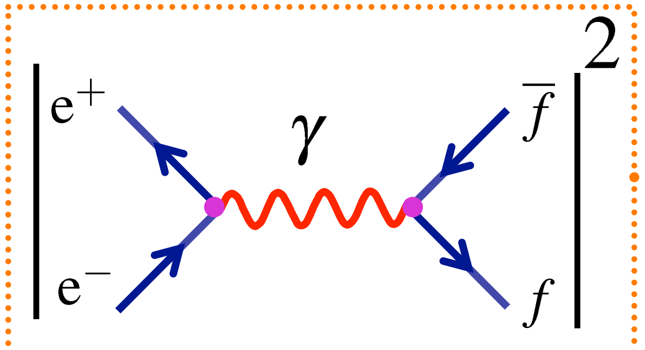
★ **1989-1995: Electron-Positron collisions at $\sqrt{s}$ = 91.2 GeV**
  - **17 Million Z bosons detected**

★ **1996-2000: Electron-Positron collisions at $\sqrt{s}$ = 161-208 GeV**
  - **30000 W^+W^- events detected**

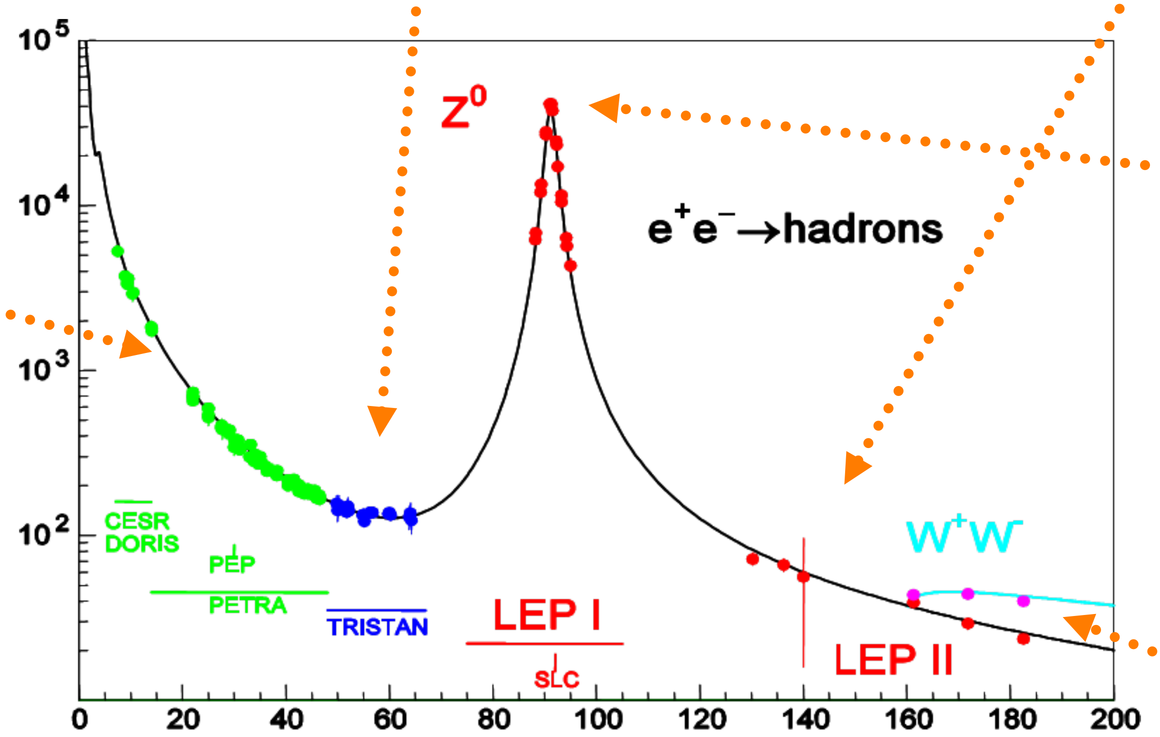
ee-Annihilation

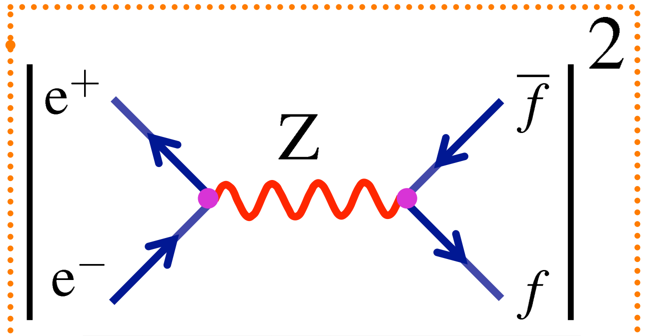
In general e^+e^- annihilation involves both photon and Z exchange : + interference





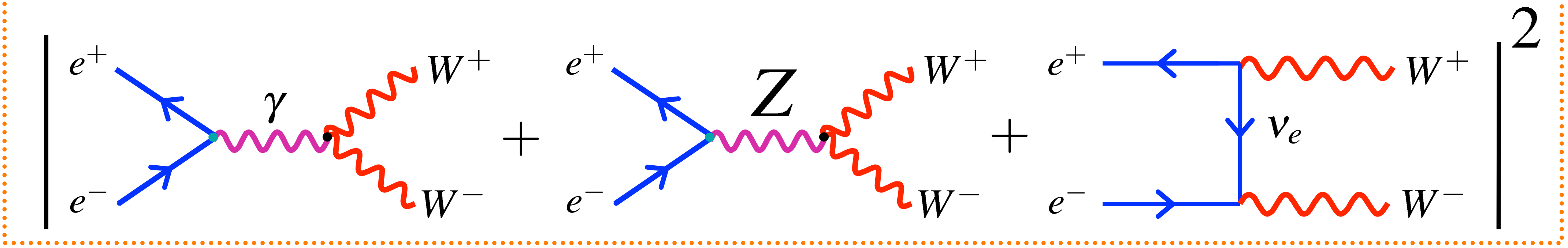
Well below Z: photon exchange dominant





At Z resonance: Z exchange dominant

High energies: WW production



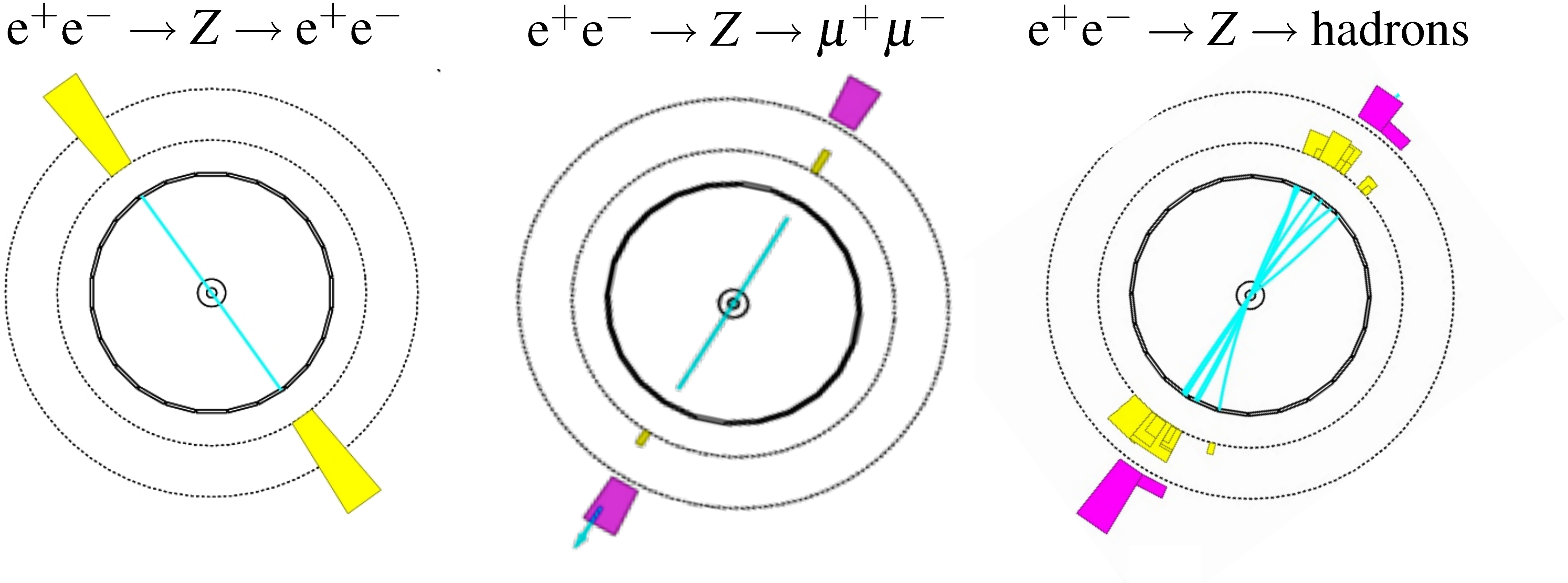
Cross Section Measurement

- ★ **At Z resonance mainly observe four types of event:**

$$\mathrm{e}^+\mathrm{e}^- \rightarrow Z \rightarrow \mathrm{e}^+\mathrm{e}^- \qquad \mathrm{e}^+\mathrm{e}^- \rightarrow Z \rightarrow \mu^+\mu^- \qquad \mathrm{e}^+\mathrm{e}^- \rightarrow Z \rightarrow \tau^+\tau^-$$

$$\mathrm{e}^+\mathrm{e}^- \rightarrow Z \rightarrow q\bar{q} \rightarrow \mathrm{hadrons}$$

- ★ **Each has a distinct topology in the detectors, e.g.**



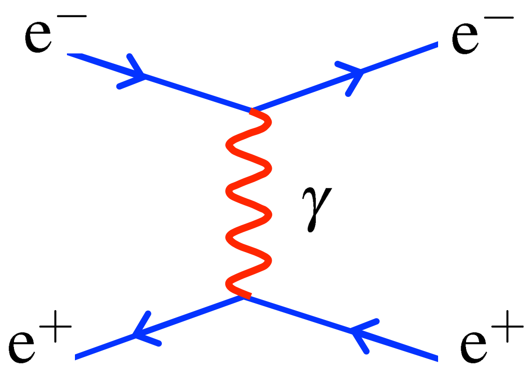
$\mathrm{e}^+\mathrm{e}^- \rightarrow Z \rightarrow \mathrm{e}^+\mathrm{e}^-$ $\qquad$ $\mathrm{e}^+\mathrm{e}^- \rightarrow Z \rightarrow \mu^+\mu^-$ $\qquad$ $\mathrm{e}^+\mathrm{e}^- \rightarrow Z \rightarrow \mathrm{hadrons}$

- ★ **To work out cross sections, first count events of each type**
- ★ **Then need to know "integrated luminosity" of colliding beams, i.e. the relation between cross-section and expected number of interactions**

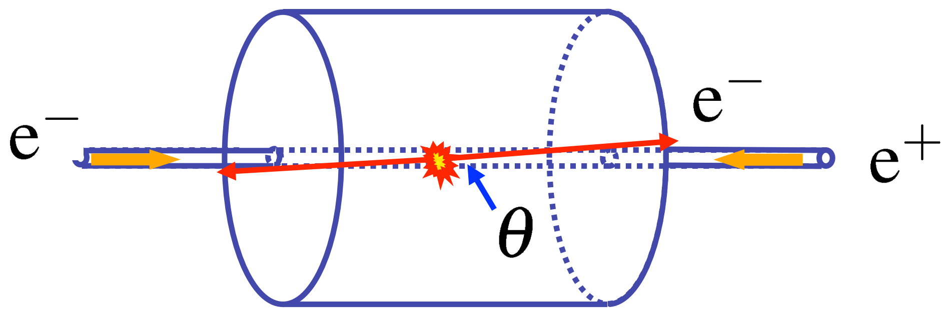
$$N_{\mathrm{events}} = \mathscr{L}\sigma$$

Cross Section Measurement

- ★ To calculate the integrated luminosity need to know numbers of electrons and positrons in the colliding beams and the exact beam profile
  - very difficult to achieve with precision of better than 10%
- ★ Instead "normalise" using another type of event:



- ♦ Use the QED Bhabha scattering process
- ♦ QED, so cross section can be calculated very precisely
- ♦ Very large cross section – small statistical errors
- ♦ Reaction is very forward peaked – i.e. the electron tends not to get deflected much



$$\frac{d\sigma}{d\Omega} \propto \frac{1}{q^4} \propto \frac{1}{\sin^4\theta/2} \Rightarrow \boxed{\frac{d\sigma}{d\theta} \propto \frac{1}{\theta^3}}$$

Photon propagator

e.g. see handout 5

- ♦ Count events where the electron is scattered in the very forward direction

$$N_{\text{Bhabha}} = \mathscr{L}\sigma_{\text{Bhabha}} \Rightarrow \mathscr{L}$$

σ_{Bhabha} known from QED calc.

- ★ Hence all other cross sections can be expressed as

$$\boxed{\sigma_i = \frac{N_i}{N_{\text{Bhabha}}}\sigma_{\text{Bhabha}}} \Rightarrow$$

Cross section measurements Involve just event counting !

The Z-lineshape

★ **Measurements of the Z resonance lineshape determine:**

- m_Z : peak of the resonance
- Γ_Z : FWHM of resonance
- Γ_f : Partial decay widths
- N_ν : Number of light neutrino generations

★ Measure cross sections to different final states versus C.o.M. energy $\sqrt{s}$

★ Starting from

$$\sigma(e^+e^- \to Z \to f\overline{f}) = \frac{12\pi}{m_Z^2}\frac{s}{(s-m_Z^2)^2+m_Z^2\Gamma_Z^2}\Gamma_{ee}\Gamma_{ff} \qquad (3)$$

maximum cross section occurs at $\sqrt{s}=m_Z$ with peak cross section equal to

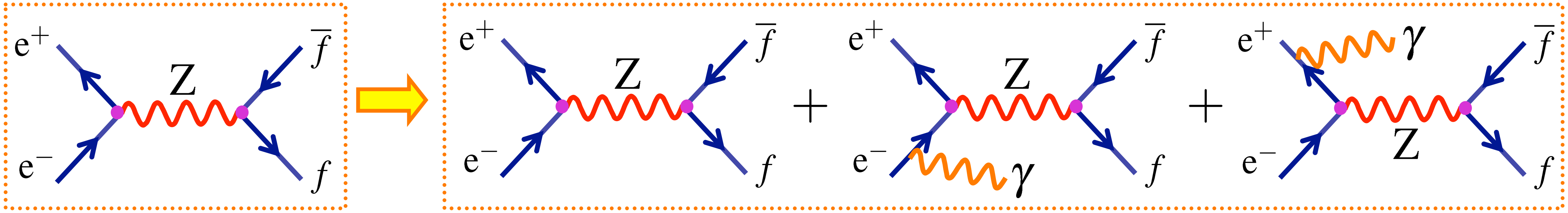
$$\boxed{\sigma^0_{f\overline{f}} = \frac{12\pi}{m_Z^2}\frac{\Gamma_{ee}\Gamma_{ff}}{\Gamma_Z^2}}$$

★ Cross section falls to half peak value at $\sqrt{s} \approx m_z \pm \frac{\Gamma_Z}{2}$ which can be seen immediately from eqn. (3)

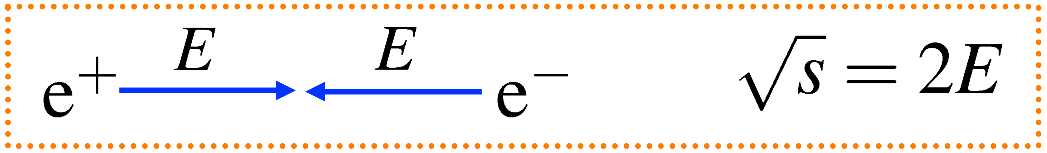
★ Hence $\Gamma_Z = \frac{\hbar}{\tau_Z} = \text{FWHM of resonance}$

The Z-lineshape

- ★ In practise, it is not that simple, QED corrections distort the measured line-shape
- ★ One particularly important correction: initial state radiation (**ISR**)

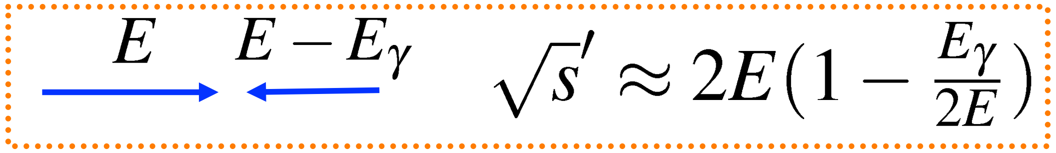


- ★ Initial state radiation reduces the centre-of-mass energy of the e^+e^- collision



$$\sqrt{s} = 2E$$

becomes



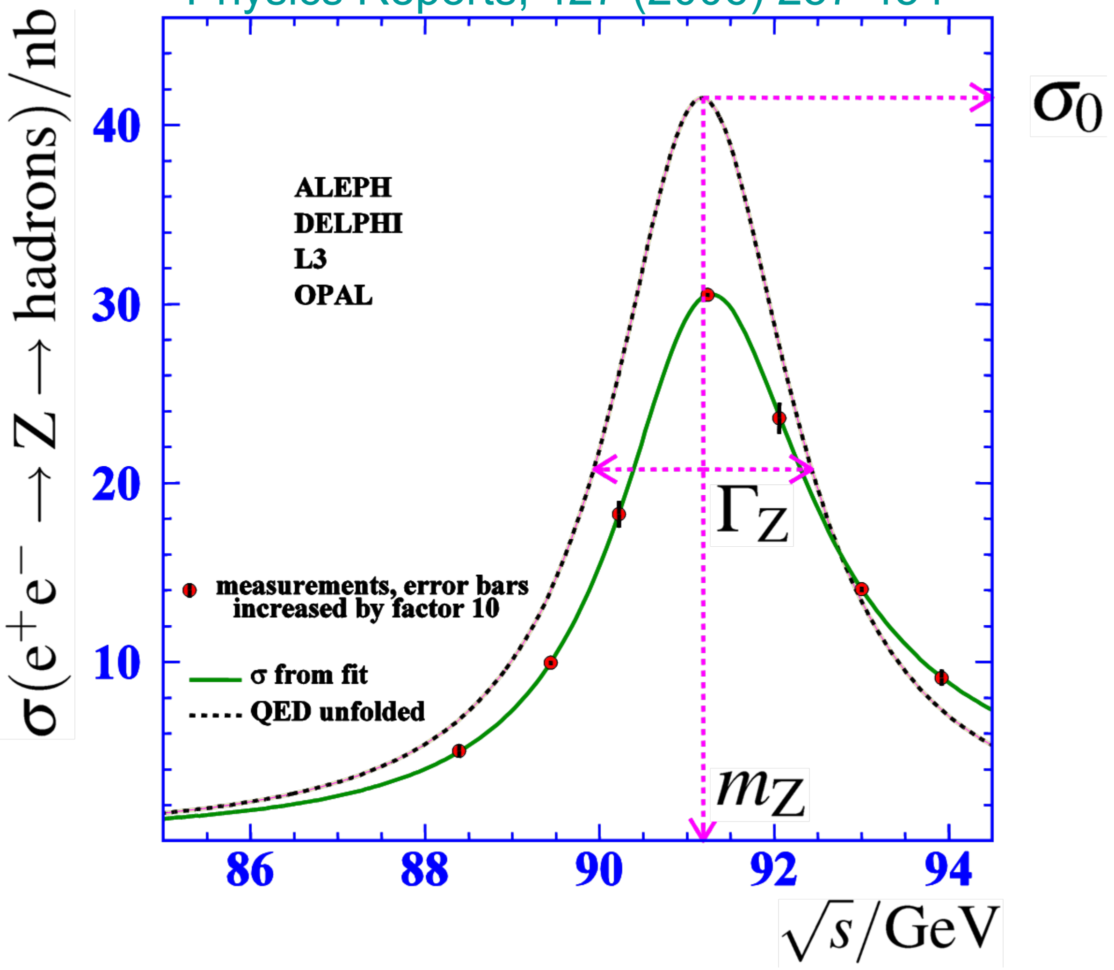
$$\sqrt{s'} \approx 2E\left(1 - \frac{E_\gamma}{2E}\right)$$

- ★ Measured cross section can be written:

$$\sigma_{\text{meas}}(E) = \int \sigma(E')f(E',E)\mathrm{d}E'$$

Probability of e+e- colliding with C.o.M. energy **E'** when C.o.M energy before radiation is **E**

- ★ Fortunately can calculate $f(E',E)$ very precisely, just QED, and can then obtain Z line-shape from measured cross section



Physics Reports, 427 (2006) 257-454

The Z-lineshape

★ **In principle the measurement of m_Z and Γ_Z is rather simple: run accelerator at different energies, measure cross sections, account for ISR, then find peak and FWHM**

$$\boxed{m_Z = 91.1875 \pm 0.0021\,\mathrm{GeV}}$$

$$\boxed{\Gamma_Z = 2.4952 \pm 0.0023\,\mathrm{GeV}}$$

★ **0.002 % measurement of m_Z !**

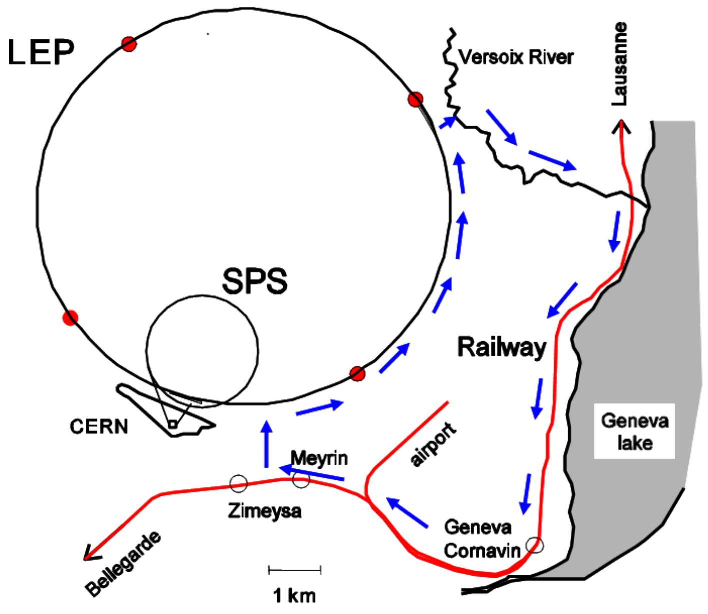
★ **To achieve this level of precision – need to know energy of the colliding beams to better than 0.002 % : sensitive to unusual systematic effects…**

Moon:

- **As the moon orbits the Earth it distorts the rock in the Geneva area very slightly !**
- **The nominal radius of the accelerator of 4.3 km varies by ±0.15 mm**
- **Changes beam energy by ~10 MeV : need to correct for tidal effects !**

Trains:

- **Leakage currents from the TGV railway line return to Earth following the path of least resistance.**
- **Travelling via the Versoix river and using the LEP ring as a conductor.**
- **Each time a TGV train passed by, a small current circulated LEP slightly changing the magnetic field in the accelerator**
- **LEP beam energy changes by ~10 MeV**



Number of Generations

★ **Total decay width measured from Z line-shape:** $\Gamma_Z = 2.4952 \pm 0.0023\,\mathrm{GeV}$

★ **If there were an additional 4th generation would expect** $Z \rightarrow \nu_4 \overline{\nu}_4$ **decays even if the charged leptons and fermions were too heavy (i.e. > $m_Z/2$)**

★ **Total decay width is the sum of the partial widths:**

$$\Gamma_Z = \Gamma_{ee} + \Gamma_{\mu\mu} + \Gamma_{\tau\tau} + \Gamma_{\text{hadrons}} + \Gamma_{\nu_1\nu_1} + \Gamma_{\nu_2\nu_2} + \Gamma_{\nu_3\nu_3} + ?$$

★ **Although don't observe neutrinos,** $Z \rightarrow \nu\overline{\nu}$ **decays affect the Z resonance shape for all final states**

★ **For all other final states can determine partial decay widths from peak cross sections:**

$$\sigma^0_{f\overline{f}} = \frac{12\pi}{m_Z^2}\frac{\Gamma_{ee}\Gamma_{ff}}{\Gamma_Z^2}$$

★ **Assuming lepton universality:**

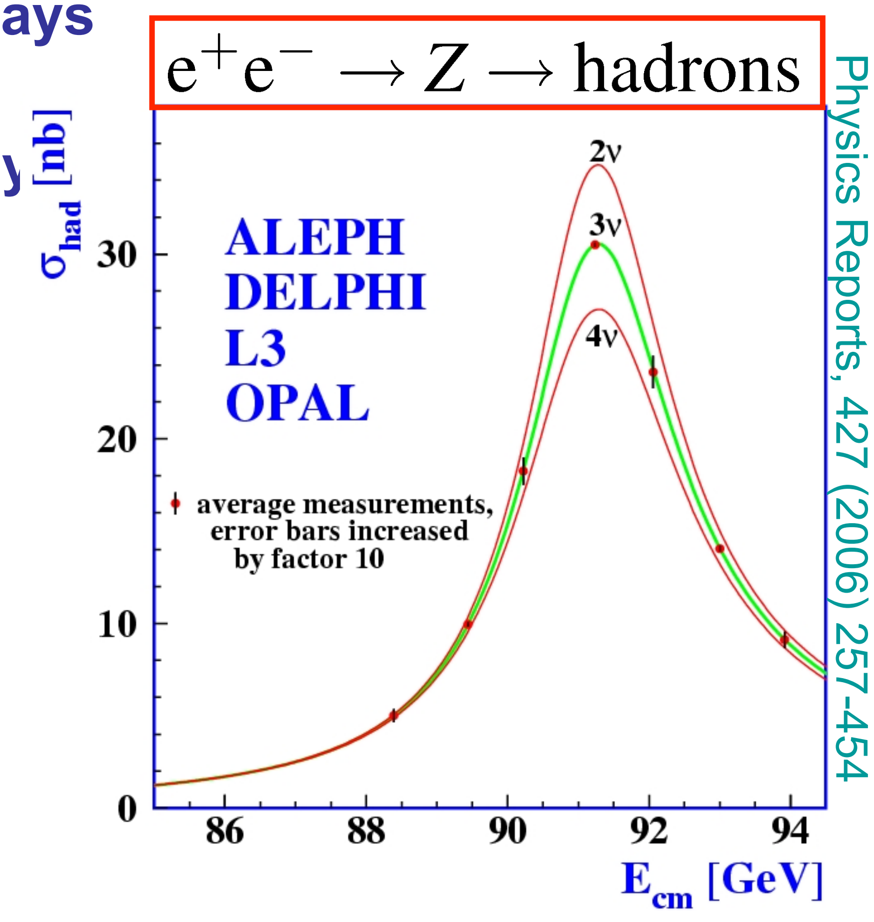
$$\Gamma_Z = 3\Gamma_{\ell\ell} + \Gamma_{\text{hadrons}} + \boxed{N_\nu}\Gamma_{\nu\nu}$$

measured from Z lineshape

measured from peak cross sections

calculated, e.g. question 26

$$\boxed{N_\nu = 2.9840 \pm 0.0082}$$



Physics Reports, 427 (2006) 257-454

★ **ONLY 3 GENERATIONS** **(unless a new 4th generation neutrino has very large mass)**

Forward-Backward Asymmetry

- ★ **We obtained the expression for the differential cross section:**

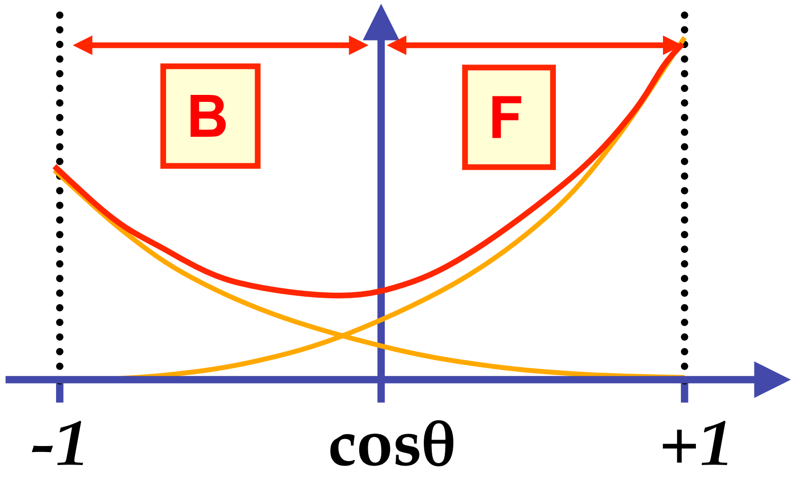
$$\langle|M_{fi}|\rangle^2 \propto [(c_L^e)^2+(c_R^e)^2][(c_L^\mu)^2+(c_R^\mu)^2](1+\cos^2\theta)+[(c_L^e)^2-(c_R^e)^2][(c_L^\mu)^2-(c_R^\mu)^2]\cos\theta$$

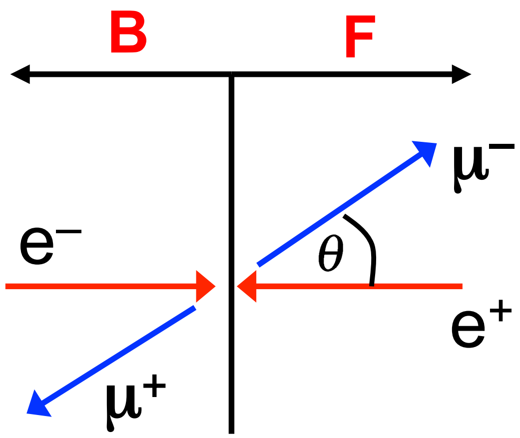
- ★ **The differential cross sections is therefore of the form:**

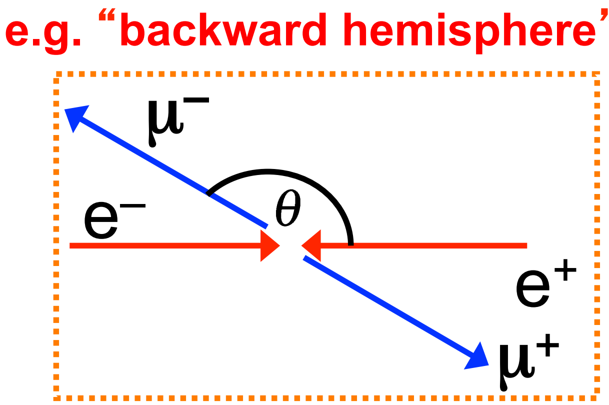
$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \kappa\times[A(1+\cos^2\theta)+B\cos\theta] \quad \begin{cases} A = [(c_L^e)^2+(c_R^e)^2][(c_L^\mu)^2+(c_R^\mu)^2] \\ B = [(c_L^e)^2-(c_R^e)^2][(c_L^\mu)^2-(c_R^\mu)^2] \end{cases}$$

- ★ **Define the FORWARD and BACKWARD cross sections in terms of angle incoming electron and out-going particle**

$$\boxed{\sigma_F \equiv \int_0^1 \frac{\mathrm{d}\sigma}{\mathrm{d}\cos\theta}\mathrm{d}\cos\theta} \qquad \boxed{\sigma_B \equiv \int_{-1}^0 \frac{\mathrm{d}\sigma}{\mathrm{d}\cos\theta}\mathrm{d}\cos\theta}$$



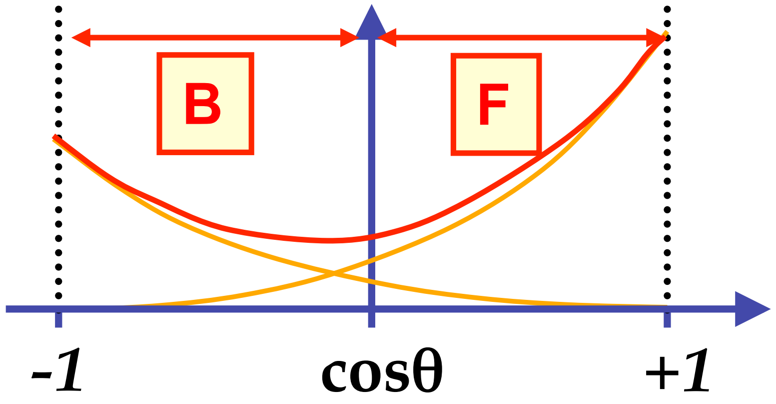




Forward-Backward Asymmetry

★ The level of asymmetry about **cosθ=0** is expressed in terms of the Forward-Backward Asymmetry

$$A_{\mathrm{FB}} = \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B}$$



- Integrating equation (1):

$$\sigma_F = \kappa \int_0^1 [A(1+\cos^2\theta) + B\cos\theta]\mathrm{d}\cos\theta = \kappa\int_0^1 [A(1+x^2)+Bx]\mathrm{d}x = \kappa\left(\frac{4}{3}A + \frac{1}{2}B\right)$$

$$\sigma_B = \kappa \int_{-1}^0 [A(1+\cos^2\theta) + B\cos\theta]\mathrm{d}\cos\theta = \kappa\int_{-1}^0 [A(1+x^2)+Bx]\mathrm{d}x = \kappa\left(\frac{4}{3}A - \frac{1}{2}B\right)$$

★ Which gives:

$$A_{\mathrm{FB}} = \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B} = \frac{B}{(8/3)A} = \frac{3}{4}\left[\frac{(c_L^e)^2 - (c_R^e)^2}{(c_L^e)^2 + (c_R^e)^2}\right]\cdot\left[\frac{(c_L^\mu)^2 - (c_R^\mu)^2}{(c_L^\mu)^2 + (c_R^\mu)^2}\right]$$

★ This can be written as

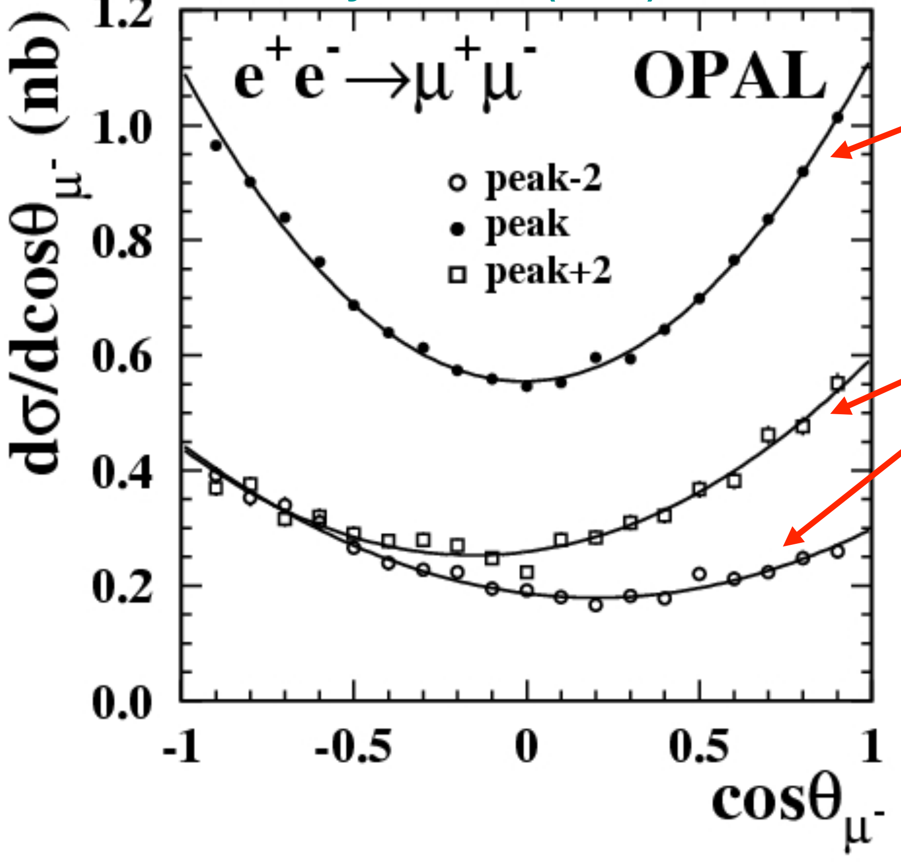
$$\boxed{A_{\mathrm{FB}} = \frac{3}{4}A_e A_\mu}$$ with $$\boxed{A_f \equiv \frac{(c_L^e)^2 - (c_R^e)^2}{(c_L^e)^2 + (c_R^e)^2} = \frac{2c_V^f c_A^f}{(c_V^f)^2 + (c_A^f)^2}} \quad (4)$$

★ Observe a non-zero asymmetry because the couplings of the Z to LH and RH particles are different. Contrast with QED where the couplings to LH and RH particles are the same (parity is conserved) and the interaction is FB symmetric

Measuring AFB

- ★ **Forward-backward asymmetries can only be measured for final states where the charge of the fermion can be determined, e.g.** $\mathrm{e}^+\mathrm{e}^- \to Z \to \mu^+\mu^-$

OPAL Collaboration, Eur. Phys. J. C19 (2001) 587-651.



Because $\sin^2\theta_w \approx 0.25$, the value of A_{FB} for leptons is almost zero

For data above and below the peak of the Z resonance interference with $\mathrm{e}^+\mathrm{e}^- \to \gamma \to \mu^+\mu^-$ leads to a larger asymmetry

★**LEP data combined:**

$$A_{FB}^{0,e} = 0.0145 \pm 0.0025$$

$$A_{FB}^{0,\mu} = 0.0169 \pm 0.0013$$

$$A_{FB}^{0,\tau} = 0.0188 \pm 0.0017$$

- ★ **To relate these measurements to the couplings uses** $A_{\mathrm{FB}} = \frac{3}{4}A_e A_\mu$
- ★ **In all cases asymmetries depend on** A_e
- ★ **To obtain** A_e **could use** $A_{FB}^{0,e} = \frac{3}{4}A_e^2$

The Weak Mixing Angle

★ **From LEP :** $A_{FB}^{0,f} = \frac{3}{4} A_e A_f$

★ **From SLC :** $A_{LR} = A_e$

$$\left.\begin{matrix} A_{FB}^{0,f} = \frac{3}{4} A_e A_f \\ A_{LR} = A_e \end{matrix}\right\} \quad A_e, A_\mu, A_\tau, \ldots$$

Putting everything together ⇨

$$\boxed{\begin{aligned} A_e &= 0.1514 \pm 0.0019 \\ A_\mu &= 0.1456 \pm 0.0091 \\ A_\tau &= 0.1449 \pm 0.0040 \end{aligned}}$$

includes results from other measurements

with $$A_f \equiv \frac{2 c_V^f c_A^f}{(c_V^f)^2 + (c_A^f)^2} = 2 \frac{c_V / c_A}{1 + (c_V / c_A)^2}$$

★ **Measured asymmetries give ratio of vector to axial-vector Z coupings.**

★ **In SM these are related to the weak mixing angle**

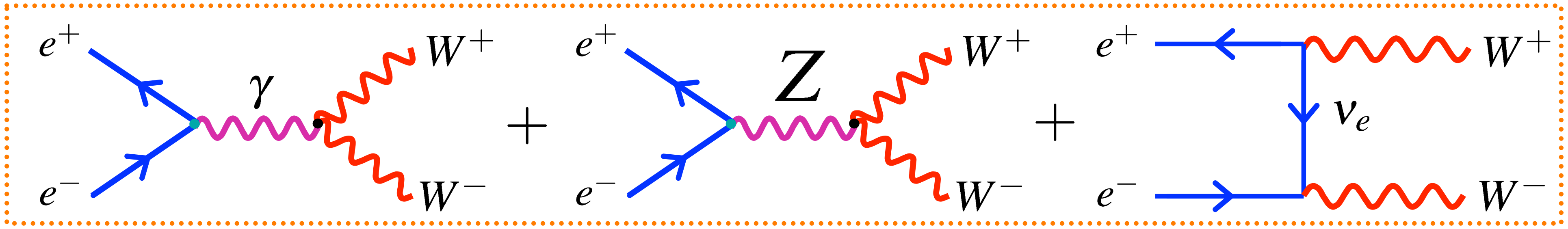
$$\frac{c_V}{c_A} = \frac{I_W^3 - 2Q \sin^2 \theta_W}{I_W^3} = 1 - \frac{2Q}{I_3} \sin^2 \theta_W = 1 - 4|Q| \sin^2 \theta_W$$

★ **Asymmetry measurements give precise determination of** $\sin^2 \theta_W$

$$\boxed{\sin^2 \theta_W = 0.23154 \pm 0.00016}$$

W-production

- ★ **From 1995-2000 LEP operated above the threshold for W-pair production**
- ★ **Three diagrams "CC03" are involved**

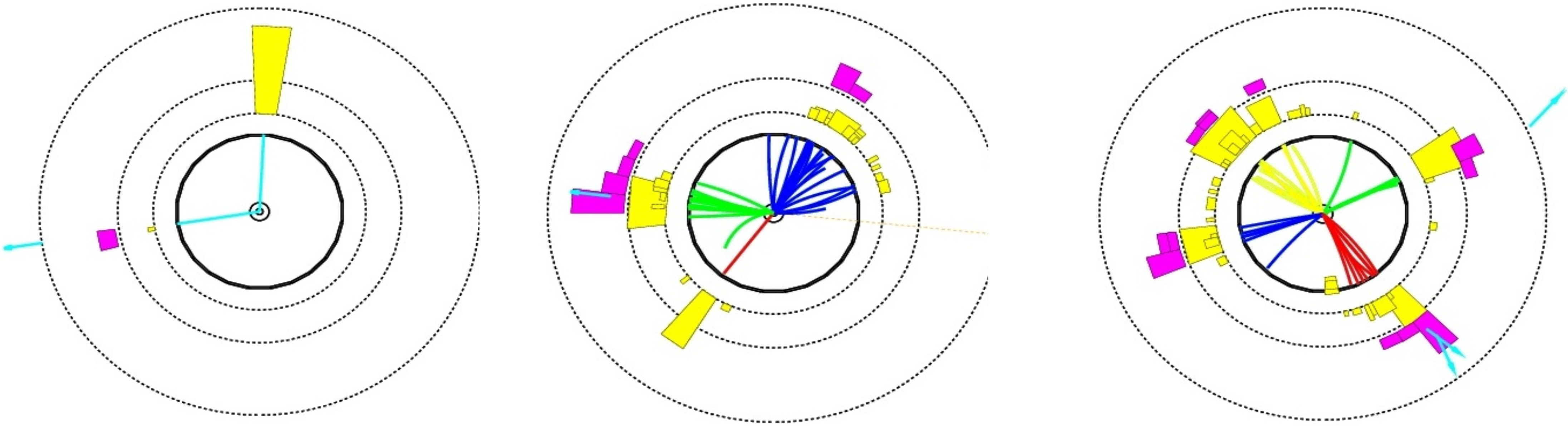


- ★ **W bosons decay (p.459) either to leptons or hadrons with branching fractions:**

$$Br(W^- \rightarrow \text{hadrons}) \approx 0.67 \qquad Br(W^- \rightarrow \mathrm{e}^-\overline{\nu}_\mathrm{e}) \approx 0.11$$

$$Br(W^- \rightarrow \mu^-\overline{\nu}_\mu) \approx 0.11 \qquad Br(W^- \rightarrow \tau^-\overline{\nu}_\tau) \approx 0.11$$

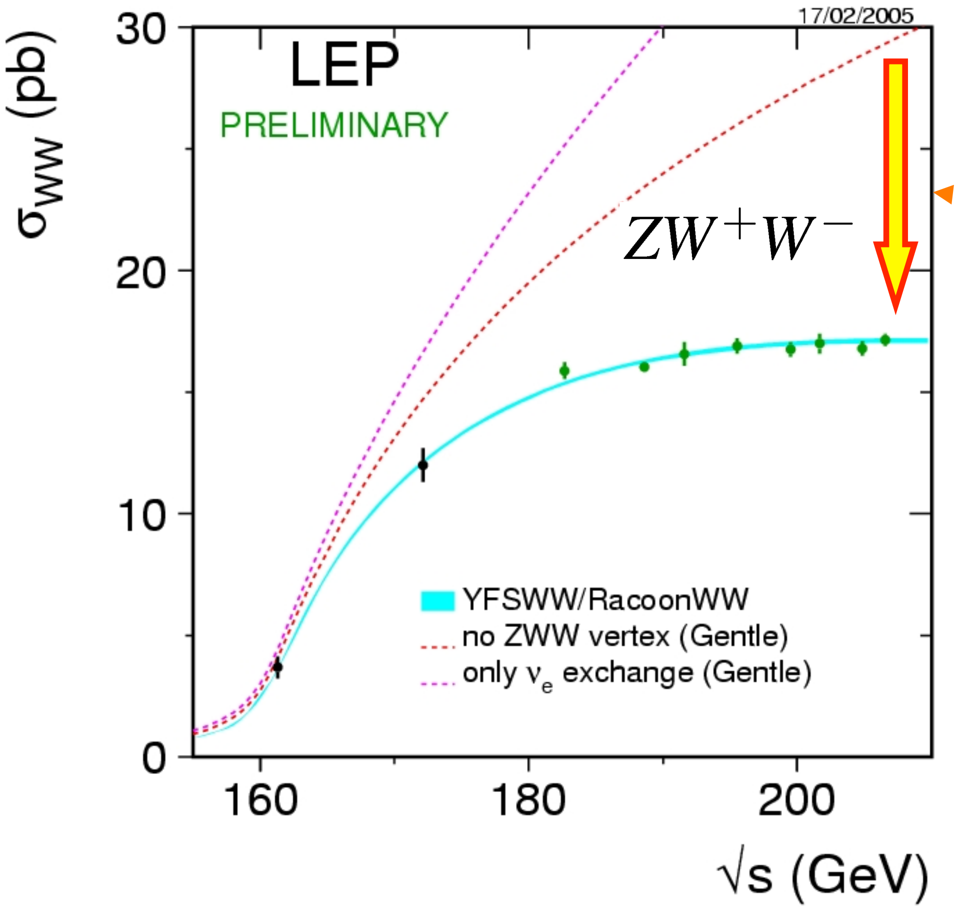
- ★ **Gives rise to three distinct topologies**



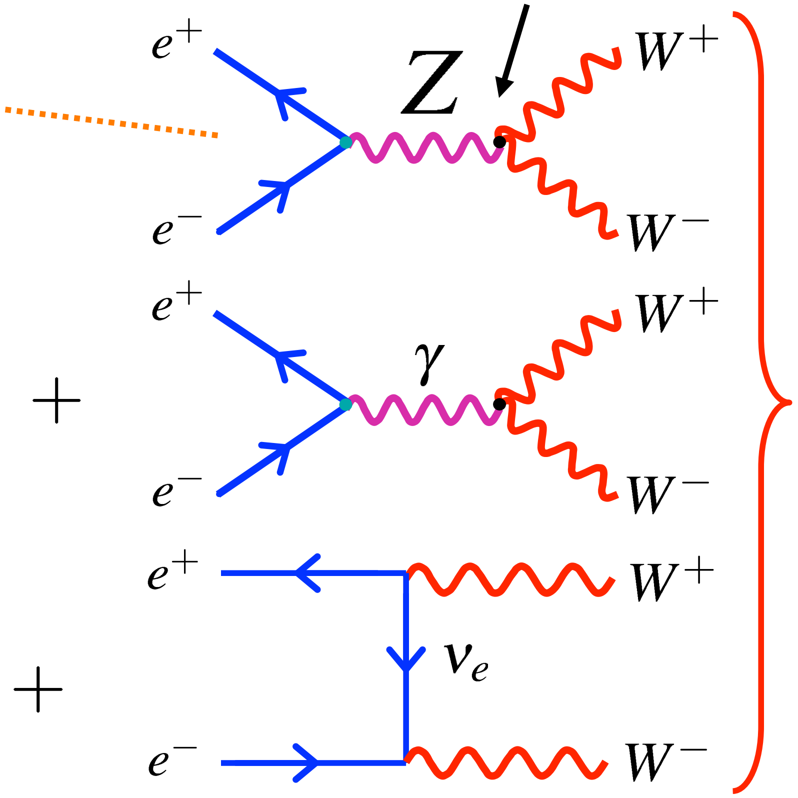
$\boxed{W^+W^- \rightarrow \ell^+\nu\ell^-\overline{\nu}}$ $\boxed{W^+W^- \rightarrow q\overline{q}\ell\nu}$ $\boxed{W^+W^- \rightarrow q\overline{q}q\overline{q}}$

W-production

- ★ Measure cross sections by **counting events** and normalising to low angle Bhabha scattering events



- ★ Data consistent with SM expectation
- ★ Provides a direct test of ZW^+W^- vertex



- ★ Recall that without the Z diagram the cross section violates unitarity
- ★ Presence of Z fixes this problem

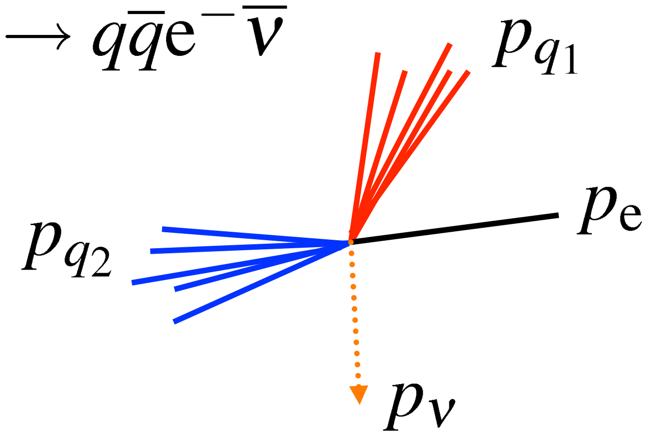
W-Production

★ **Unlike $e^+e^- \to Z$, the process $e^+e^- \to W^+W^-$ is not a resonant process**

⇨ **Different method to measure W-boson Mass**

- **Measure energy and momenta of particles produced in the W boson decays, e.g.**

$$W^+W^- \to q\bar{q}e^-\bar{\nu}$$

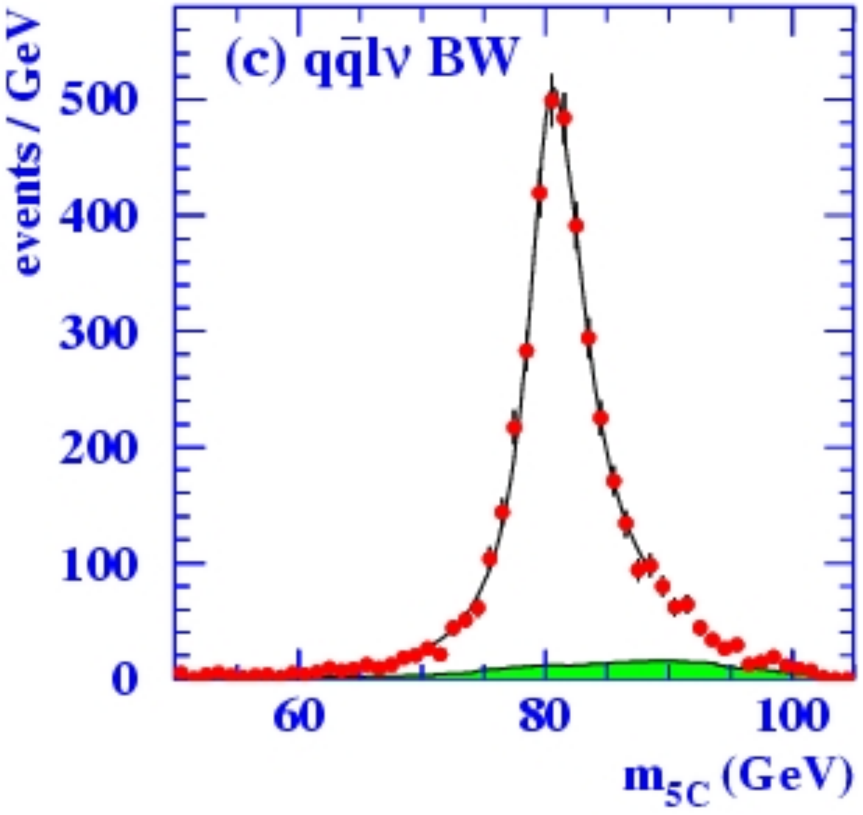


- **Neutrino four-momentum from energy-momentum conservation !**

$$p_{q_1} + p_{q_2} + p_e + p_\nu = (\sqrt{s}, 0)$$

- **Reconstruct masses of two W bosons**

$$M_+^2 = E^2 - \vec{p}^2 = (p_{q_1} + p_{q_2})^2$$
$$M_-^2 = E^2 - \vec{p}^2 = (p_e + p_\nu)^2$$



$$\approx \frac{1}{2}(M_+ + M_-)$$

★ **Peak of reconstructed mass distribution gives**

$$\boxed{m_W = 80.376 \pm 0.033\,\text{GeV}}$$

★ **Width of reconstructed mass distribution gives:**

$$\boxed{\Gamma_W = 2.196 \pm 0.083\,\text{GeV}}$$

Does not include measurements from Tevatron at Fermilab