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General Relativity
Sheet 1

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Exercise 1 [Classical (Newton/Poisson) Gravity and Galilean Transformations]

The Galilean transformations are given by

$$\mathbf{x} \to \mathbf{x}' = R\,\mathbf{x} + \mathbf{v}t + \mathbf{x_0}, \quad t \to t + t_0, \quad R \in O(3) = \text{const}, \mathbf{v} = \text{const}$$

Show that both

(i) the Newton equations of motion for N self-gravitating point masses (N-body interaction)

$$m_k \frac{\mathrm{d}^2 \mathbf{x}_k}{\mathrm{d}t^2} = \sum_{l \neq k} f_{kl}\left(|\mathbf{x}_k - \mathbf{x}_l|\right)\left(\mathbf{x}_k - \mathbf{x}_l\right), \quad k = 1, \dots, N \quad \text{and,}$$

(ii) the classical gravity field equation

$$\nabla^2 \Phi\left(\mathbf{x}\right) = 4\pi G \rho\left(\mathbf{x}\right),$$

are invariant under Galilean transformations.

Exercise 2 [The Motion of Test Particles in Special Relativity]

The equations of motion for a free particle in special relativity can be derived from the action

$$S = -\alpha \int \mathrm{d}s = -\alpha \int \mathrm{d}\tau \sqrt{\eta_{\mu\nu} \frac{\mathrm{d}x^\mu}{\mathrm{d}\tau} \frac{\mathrm{d}x^\nu}{\mathrm{d}\tau}},$$

where $\alpha = mc$ is a positive constant, and $\mathrm{d}s$ the invariant spacetime interval. Derive the equations of motion.

Exercise 3 [Lorentz Transformations]

(i) If $\Lambda^\alpha{}_\beta$ and $\tilde{\Lambda}^\gamma{}_\delta$ are Lorentz transformations, show that $\left(\Lambda\tilde{\Lambda}\right)^\mu{}_\nu$ is also a Lorentz transformation.

(ii) If $\Lambda^\mu{}_\nu$ and $\tilde{\Lambda}^\mu{}_\nu$ are Lorentz boosts in the x direction with velocities v and $\tilde{v}$, compute the velocity of the boost $\left(\Lambda\tilde{\Lambda}\right)^\rho{}_\sigma$.