PHY213 - KT II
Exercise Sheet 6

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Exercise 1: Unstable mesons

A quantum state composed composed of a superposition of two flavour eigenstates $|B^0\rangle$ and $|\bar{B^0}\rangle$ at time t can be written as:

$$|\psi(t)\rangle = a(t)|B^0\rangle + b(t)|\bar{B^0}\rangle$$

Show that the current of probability associated to the evolution of this state is conserved if the hamiltonian $\mathcal{H}$ is self adjoint.

Exercise 2: Oscillating neutral mesons

Assume that the hamiltonian of the system $\mathcal{H}$ is non diagonal in this basis

$$\mathcal{H} = \begin{bmatrix} M & M_{12} \\ M_{12}^* & M \end{bmatrix} + \frac{i}{2}\begin{bmatrix} \Gamma & \Gamma_{12} \\ \Gamma_{12}^* & \Gamma \end{bmatrix}$$

Show that they eigenvalues associated to the basis that diagonalizes the hamiltonian are given by

$$E_{H,L} = \left(M \pm \frac{\Delta M}{2}\right) - \frac{i}{2}\left(\Gamma \mp \frac{\Delta\Gamma}{2}\right)$$

Where

$$\Delta M = 2\mathrm{Re}\sqrt{\left(M_{12} - \frac{i}{2}\Gamma_{12}\right)\left(M_{12}^* - \frac{i}{2}\Gamma_{12}^*\right)}$$
$$\Delta\Gamma = 4\mathrm{Im}\sqrt{\left(M_{12} - \frac{i}{2}\Gamma_{12}\right)\left(M_{12}^* - \frac{i}{2}\Gamma_{12}^*\right)}$$

Exercise 3: Time evolution of the flavour eigenstates

a) The mass eigenstates can be written as a linear superposition of the flavour eigenstates as

$$|B_L\rangle = p|B^0\rangle + q|B^0\rangle \qquad |B_H\rangle = p|B^0\rangle - q|B^0\rangle$$

With $p, q \in \mathbb{C}$ and $|q|^2 + |p|^2 = 1$

Using the fact that the mass hamiltonian is is diagonal in this new basis, derive the time evolution of the flavour eigenstates $|B^0(t)\rangle$ and $|\bar{B}^0(t)\rangle$

b) Use the time dependence of the flavour eigenbasis derived in point a) to write the probabilities of survival and oscillation of the neutral mesons

$$P(B^0, t=0 \rightarrow B^0, t=t) = |\langle B^0|B^0(t)\rangle|^2 \qquad P(B^0, t=0 \rightarrow \bar{B}^0, t=t) = |\langle \bar{B}^0|B^0(t)\rangle|^2$$
$$P(\bar{B}^0, t=0 \rightarrow \bar{B}^0, t=t) = |\langle \bar{B}^0|\bar{B}^0(t)\rangle|^2 \qquad P(\bar{B}^0, t=0 \rightarrow \bar{B}^0, t=t) = |\langle \bar{B}^0|\bar{B}^0(t)\rangle|^2$$

Exercise 4: The B_d-$\bar{B}_d$ system

A time-dependent oscillation asymmetry can be defined as

$$A(t) = \frac{N(B^0, t=0 \rightarrow B^0, t=t) - N(B^0, t=0 \rightarrow \bar{B}^0, t=t)}{(B^0, t=0 \rightarrow B^0, t=t) + N(B^0, t=0 \rightarrow \bar{B}^0, t=t)}$$

If we consider the $B_d - \bar{B}_d$ system it is characterised by $\Delta\Gamma \sim 0$ and in good approximation $|p/q| \sim 1$ Show that the time dependent asymmetry becomes

$$A(t) = cos(\Delta M t)$$

In this limit