Role of electromagnetic coupling in the low-field phase diagram of Bi$_{2.15}$Sr$_{1.85}$CaCu$_2$O$_{8+\delta}$

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A detailed study has been made of a transition in the flux lattice of the superconductor Bi$_{2.15}$Sr$_{1.85}$CaCu$_2$O$_{8+\delta}$ using the technique of muon-spin rotation. The results are in excellent agreement with a recent theoretical analysis on the low-field phase diagram of highly anisotropic superconductors. In particular, the important role of electromagnetic interactions between vortex segments is demonstrated. The form of the phase boundary suggests that the transition may be identified with a simultaneous melting and decoupling of the vortices in adjacent layers. [S0163-1829(97)05706-8]

The behavior of flux vortices in high temperature superconductors leads to many new phenomena not encountered in conventional superconductors.\textsuperscript{1–5} Techniques such as small-angle neutron scattering (SANS), NMR, and muon-spin rotation (μSR), which microscopically probe the vortex state, have been crucial to increasing the understanding of these systems.\textsuperscript{6–12} Recently studies of Bi$_{2.15}$Sr$_{1.85}$CaCu$_2$O$_{8+\delta}$ (BSCCO) have attracted much attention, since the extreme superconducting properties of this material lead to very exotic vortex behavior.\textsuperscript{5,13,10–12} In particular the very large superconducting anisotropy of BSCCO leads to vortex lines which are highly flexible, which in turn makes them extremely susceptible to thermally induced and pinning-induced disorder.\textsuperscript{5,13,10–12} A more appropriate description of a vortex line in this material is that of a string of two-dimensional “pancake” vortices, each confined to a pair of CuO$_2$ planes of the crystal but weakly connected via interlayer interactions.\textsuperscript{14} The precise nature of this interlayer coupling is an important question. It is usually assumed to arise from Josephson tunnelling currents, and the normally much weaker electromagnetic coupling is often ignored. It has recently been suggested by Blatter \textit{et al.}\textsuperscript{13} that in BSCCO the role of electromagnetic coupling may not only be significant but may also be dominant over much of the low-field phase diagram.

The degree of superconducting anisotropy may be described by the quantity $\gamma=\lambda_z/\lambda_{ab}$, where $\lambda_z$ and $\lambda_{ab}$ are the penetration depths for currents flowing perpendicular and parallel to the CuO$_2$ planes, respectively. In the limit $\gamma=\infty$, Josephson coupling vanishes and the interlayer coupling between pancake vortices is purely electromagnetic in origin. The regime where Josephson coupling is relevant is given by $\lambda_{ab}/\gamma s$, where $s$ is the layer spacing.\textsuperscript{13} In contrast, for BSCCO $\lambda_{ab}(0)=1800$ Å,\textsuperscript{10} $\gamma\sim 150$,\textsuperscript{15} and $s=15$ Å so that $\lambda_{ab}(0)/\gamma s$, which suggests that electromagnetic coupling should control the vortex behavior over much of the phase diagram.\textsuperscript{13} The rigidity of a vortex line is controlled by the tilt modulus of the lattice which contains terms arising from both a nonlocal contribution to the tilt energy and also two “single vortex” contributions. One of these single-vortex terms, which is often neglected, arises from the electromagnetic interaction.\textsuperscript{4,13} This contribution is highly dispersive with a corresponding stiffness that goes as $1/\lambda_{ab}^2 k_z^2$, where $k_z$ is the wave vector for displacements perpendicular to the vortex direction $z$.

In previous μSR and SANS experiments we have demonstrated the existence of a vortex-line lattice,\textsuperscript{10,11} in which the vortices are subject to strong thermally induced short wavelength fluctuations, despite their long wavelength stiffness. This strongly dispersive behavior, together with the fact that $\lambda_{ab}(0)/\gamma s$, both suggest a system at the limit of vanishing Josephson coupling. A recent theoretical treatment of the phase diagram of highly anisotropic superconductors by Blatter \textit{et al.}\textsuperscript{13} has made detailed predictions of the differing behavior of vortices for the different types of interlayer coupling. Here we present experimental results which allow us to test in detail the predictions of this theory and
thus to address the important and long-standing question of the role of electromagnetic coupling in BSCCO.

The crystals were prepared as in Ref. 16, by annealing after growth in air at 500° and quenching in order to give a well-defined oxygen concentration and a $T_c \approx 84$ K. The measurements were performed on beamline πM3 at the Paul Scherrer Institute, Switzerland, using an experimental arrangement as described in Refs. 10 and 12. A mosaic of crystals were aligned with their $c$-axes perpendicular to an Fe$_2$O$_3$ plate and parallel to the applied field. Low momentum muons were incident on the sample with their momentum parallel to the magnetic field and with their spins polarized perpendicular to the field. The purpose of the Fe$_2$O$_3$ plate was to rapidly depolarize, outside of the observable time window, any muons not hitting the sample. In this way background signals in the $\mu$SR spectra were avoided.

In a $\mu$SR experiment the spin-polarized muons precess in the local internal fields of the mixed state of the superconductor. By measuring the distribution of precession frequencies, the probability distribution $p(B)$ of the internal field values may be determined, which in turn is intimately related to the spatial distribution of the vortex fields $B(r)$. In the light of recent theoretical developments, we have performed very detailed measurements on the vortex-lattice transition which occurs in the low-field region of the magnetic phase diagram, in order to highlight features which clarify the role of electromagnetic and Josephson coupling.

The occurrence of a transition in the flux lattice of BSCCO is readily observable using $\mu$SR. The line shape $p(B)$ is obtained from the time evolution of the muon spins via a maximum entropy technique. At $T_m(B)$ there is a sharp change in the line shape, reflecting the dramatic changes which occur in $B(r)$ as the vortex-line lattice disappears. Figure 1 illustrates the changes in a $\mu$SR line shape observable at $T_m(B)$. The skewness of the line shapes may be quantified by a parameter formed from the ratio of the third and second moments of the field probability distribution, $\alpha = \langle (\Delta B)^3 \rangle / \langle (\Delta B)^2 \rangle^{1/2}$; a distribution having a weighting towards fields higher than the average would have a positive $\alpha$, while a symmetric distribution would have an $\alpha = 0$. The value of $\alpha \approx 1.2$ observed for $T < T_m$ is in agreement with that expected for a vortex-line lattice under the measurement conditions of the experiment. The sharp change in the line shape which occurs as the vortex lines disappear thus permits the phase line $T_m(B)$, which has been identified as a vortex-lattice melting line, to be mapped out.

Figure 2 is the measured phase line for low applied fields as determined by such changes. The measured phase line may be clearly divided into two regions, separated by a crossover at a temperature which we denote as $T^m \approx 70$ K. For temperatures above and below $T^m$ the form of the curve $B_m(T)$ differs, which is more clearly demonstrated by the logarithmic plot shown in the inset of Fig. 2. It is this change in the temperature dependence $B_m(T)$ which is one of the important findings presented in this paper.

Blatter et al. have recently discussed the low-field phase diagram of layered superconductors and the role of electromagnetic coupling in controlling the behavior of the vortex lattice. Of particular relevance to BSCCO is the situation where the penetration depth $\lambda_{ab} \ll \gamma s$. For fields $B < \phi_0/\lambda_{ab}^2$, the electromagnetic coupling dominates the behavior of the vortices and hence the form of the melting line. Even though the anisotropy $\gamma$ is finite, at low temperatures the melting line is found to be independent of $\gamma$, and has the form

$$B_m^m(T) = \frac{\phi_0^3 c_s^2 s}{4 \pi \mu_0^2 k_B \beta} \frac{1}{\lambda_{ab}^4 T}.$$

where $\beta = [\ln(1+4\lambda_{ab}^2/c_s^2 a_0)]^{-1}$, $a_0$ is the lattice parameter, and $c_s$ is the Lindemann melting number which is a constant $\sim 0.1$–$0.2$. Although the vortex lattice may be dominated by the electromagnetic coupling at low temperature, it may cross over to one where Josephson coupling plays a role above a characteristic temperature $T^m = T_c$. The occurrence of a transition in the flux lattice of BSCCO is readily observable using $\mu$SR. The line shape $p(B)$ is obtained from the time evolution of the muon spins via a maximum entropy technique. At $T_m(B)$ there is a sharp change in the line shape, reflecting the dramatic changes which occur in $B(r)$ as the vortex-line lattice disappears. Figure 1 illustrates the changes in a $\mu$SR line shape observable at $T_m(B)$. The skewness of the line shapes may be quantified by a parameter formed from the ratio of the third and second moments of the field probability distribution, $\alpha = \langle (\Delta B)^3 \rangle / \langle (\Delta B)^2 \rangle^{1/2}$; a distribution having a weighting towards fields higher than the average would have a positive $\alpha$, while a symmetric distribution would have an $\alpha = 0$. The value of $\alpha \approx 1.2$ observed for $T < T_m$ is in agreement with that expected for a vortex-line lattice under the measurement conditions of the experiment. The sharp change in the line shape which occurs as the vortex lines disappear thus permits the phase line $T_m(B)$, which has been identified as a vortex-lattice melting line, to be mapped out.

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\[ -\beta [\pi \lambda_{ab}(0)/\gamma s]^{1/4}. \]

The form of the melting line is then expected to change from that of Eq. (1) to
\[ B_{m}^{em}(T) = \frac{\phi_{0}^{3} \pi \xi_{c}^{2}}{8 \pi \mu_{0} \beta k_{B} T} \frac{1}{\lambda_{ab}(T)} \]
with a corresponding crossover at \( T_{m}^{em} \). At higher fields \( B > \phi_{0}/\lambda_{ab} \) and \( a_{0} < \gamma s \), \( B_{m}^{em}(T) \) runs into the usual two-dimensional melting curve \( T_{m}^{2D} \).

According to Ref. 13, the form of the melting line for \( T < T_{m}^{em} \) is sufficient to identify the type of coupling which is dominant. For a melting-line controlled by electromagnetic coupling \( B_{m}^{em}(T) \propto \lambda_{ab}(T)/T^{1/2} \) [ignoring the weak logarithmic factor of Eq. (1)], while for one dominated by Josephson coupling \( B_{m}^{em}(T) \propto \lambda_{ab}(T)T^{-1} \). For the intermediate case \( B_{m}^{em}(T) \) the temperature dependence is somewhat weaker [Eq. (2)]. Given the data, the evidence for the significant influence of electromagnetic coupling on the low temperature melting line in BSCCO is very strong. First, for \( T < T_{m}^{em} \) the fit of Eq. (1) to the melting line is extremely good; previous \( \mu SR \) measurements indicate a two-fluid behavior of \( \lambda_{ab} = \lambda_{ab}(0)/[1 - (T/T_{c})^{4}]^{1/2} \) (Ref. 12) with \( \lambda_{ab}(0) = 1800 \text{Å}^{10} \) leaving the only remaining undetermined parameter as the Lindemann number \( c_{L} \). The fit yields \( c_{L} = 0.18 \), which is very reasonable. 1–5 The corresponding value of \( \gamma \approx 100 \) obtained from \( T_{m}^{em} \approx 70 \text{K} \) would indeed suggest that this is the correct regime \( \lambda_{ab} \approx \gamma s \). Moreover, the change of the temperature dependence of the phase line at \( T_{m}^{em} \) to a weaker \( T \) dependence of the melting line is in agreement with theoretical predictions for the behavior on entering the regime \( T_{m}^{em} < T < T_{c} \). For \( T > T_{m}^{em} \) we have fitted a phenomenological curve
\[ B_{m}^{ph}(T) = \frac{K}{\lambda_{ab}(T)}, \]
where \( K \) is a constant and \( m = 4.3 \) would correspond to Eqs. (1) and (2), respectively. Fits to the data yield a value of \( m = 2.1(1), K = 7.6 \times 10^{-14} \text{V s K}^{-1} \). That is, the exponent \( m \approx 2 \), compared to a value of \( m = 3 \) predicted by Eq. (2), so that the temperature dependence of the data in this region is in fact even weaker than that suggested by Eq. (2). To account for this we must first discuss another scenario.

A further transition of the vortex phase is expected to occur which takes the 3D bulk system into one of decoupled vortices. This occurs as strong thermal fluctuations of the pancake vortices within the individual layers causing a loss of interlayer superconducting coherence. According to Ref. 13, in the appropriate region of low fields and large anisotropy, for \( T < T_{m}^{em} \) this takes the form \( B_{m}^{em} \propto \lambda_{ab}(T)T^{-1} \) and either lies closely below or collapses onto the melting line. For \( T > T_{m}^{em} \) this dependence crosses over to \( B_{m} \propto \lambda_{ab}(T)T^{-1} \). This is precisely the change in temperature dependence which we observe in the data on going through \( T_{m}^{em} \). It therefore seems highly probable that the transition which we trace is a decoupling of a vortex lattice whose interactions are controlled largely by electromagnetic forces for \( T < T_{m}^{em} \).

We now address the important question of the relation between the melting and the decoupling lines. Within the limits of experimental uncertainty the transitions observed via \( \mu SR \), both above and below \( T_{m}^{em} \), occur at the same phase line at which SANS experiments also indicate the loss of a vortex-line lattice. 11,17 Even at low fields no further indications of any change of vortex arrangement occur in the SANS data other than at this line, where the scattered intensity disappears within a short temperature interval. 11 Given the current data this suggests that in this phase region the decoupling and melting of the lattice are either very closely separated or occur simultaneously. These transitions also coincide with the irreversibility line as determined by high-precision torque magnetometry, 18 which further suggest an associated decoupling process.

Given this interpretation we can analyze the \( \mu SR \) phase line in terms of two expressions for vortex lattice decoupling. For \( T < T_{m}^{em} \) it takes the form
\[ B_{m}^{em}(T) = \frac{\phi_{0}^{3} \pi c_{D}}{4 \pi \mu_{0} k_{B} T} \frac{1}{\lambda_{ab}(T)}, \]
which may coincide with Eq. (1). For \( T > T_{m}^{em} \)
\[ B_{m}^{em}(T) = \frac{\phi_{0}^{3} \pi c_{D}}{4 \pi \mu_{0} k_{B} T} \frac{1}{\lambda_{ab}(T)}, \]
where \( c_{D} \approx 0.1 \) is a constant which plays a role similar to the Lindemann number and allows an estimate for the decoupling line to be obtained. 13,5 Applying Eq. (4) to the data for \( T < T_{m}^{em} \) then yields the fit shown in Fig. 2 with the constant \( c_{D} \approx 0.076 \). Equating Eqs. (4) and (5) yields an estimate of \( T_{m}^{em} \sim \left[ T_{c}\left[1 - \lambda_{cr}(T_{c})\gamma s^{2}\right]^{0}\right]^{1/4} \) for \( T_{m}^{em} = 70 \text{K} \) gives \( \gamma \approx 160 \). Putting these values of \( c_{D, \gamma} \) back into Eq. (5) leaves no remaining undetermined parameters and the resulting parameter free fit is shown as the dashed line of Fig. 2, where the agreement with the data is remarkable.

We now turn to effects at higher fields, where at low temperatures a pinning crossover occurs in the vortex lattice of BSCCO at fields \( B > B_{c_{1}} \approx 65 \text{meV} \). 10,11 Above \( B_{c_{1}} \), the soft vortex lines become highly disordered along their length due to short wavelength static tilted deformations, which result from the effects of point pinning on individual or small groups of pancake vortices. 10–12 For a strongly Josephson coupled system, a crossover to a more 2D vortex arrangement is expected as the field exceeds \( B_{2D} = \phi_{0}/(s \gamma)^{2} \) due to the increasing energetic cost of shear over tilt deformations of the lattice. 1–5 However, we have recently demonstrated that in this and other systems obeying \( \lambda_{ab}(0) > \gamma s \), this crossover occurs at the higher field of \( B_{cr} = \phi_{0}/\lambda_{ab}(T)^{2} \), rather than at \( B_{2D} \). This reflects the rather dilute vortex state at these low fields in these systems. 19

Recently we performed an analysis in BSCCO of the temperature dependence of the \( \mu SR \) linewidths using a model which includes the influence of the vortex fluctuations \( \langle u^{2}\rangle(B, T) \):
\[ \langle \Delta B^{2}(T) \rangle = B^{2} \sum_{\tau_{0}+2} e^{-\tau^{2}/2}, \]
where \( B \) is the average internal flux density and \( \tau \) are the reciprocal vortex-lattice vectors. 1,2 In that work it was found necessary to use a \( \langle u^{2}\rangle(B, T) \) dominated by short wavelength excursions of the vortex positions. Here we reanalyze the
FIG. 3. Measurements taken from Ref. 12 of the μSR linewidths $\langle(\Delta B)^2\rangle(B,T)$ at fields of 10 mT (filled circles) and 30 mT (empty circles). For $T<T_{em}$, the data are no longer described by the newly derived expression for vortex fluctuations below $T_{em}$, $\langle u_{em}^2(B,T) \rangle$ (Ref. 13) (solid curves). The dashed line is the expected curve for $T_{em}<T<T_{m}$ at 10 mT, using $\langle u_{em}^2(B,T) \rangle$ (Ref. 13). The 10 mT data migrates towards this curve for $T_{em} \approx T$.

data using the appropriate expression $\langle u_{em}^2(B,T) \rangle = 8 \pi \mu_0 k_B T \beta \lambda^4 s \phi_0^2$ taken from Ref. 13. This is very similar to the previously used expression [Eq. (4) of Ref. 12] based on weakly Josephson-coupled layers, except that it no longer depends on the anisotropy $\gamma$, since the electromagnetic coupling dominates over the Josephson terms for $T<T_{em}$. Moreover, all the parameters needed to describe $\langle u_{em}^2(B,T) \rangle$ are known from independent measurements, so that following the procedure of Ref. 12, Eq. (6) may be used to describe the temperature- and field-dependent linewidth without any adjustable parameters. These are shown as solid curves in Fig. 3, which are in excellent agreement with the data in the appropriate region, $T<T_{em}, T_m$. The Lindemann number $c_L$ is taken from the fits of the decoupling-melting line $T<T_{em}$, but its influence through $\beta$ is in any case weak due to the logarithmic dependence. Finally we note that at low fields where $T_{em}$ is crossed in the vortex phase $(T_{em} < T_m)$ the form of the vortex fluctuations changes to $\langle u_{em}^2(B,T) \rangle$ due to the increasing short wavelength stiffness of the vortex lines. This leads to a second variation $\langle \Delta B^2 \rangle \times (T>T_{em})$ as given by the dashed line of Fig. 3 for 10 mT. $\langle \Delta B^2 \rangle(T)$ should then migrate towards this curve around $T_{em}$, which indeed gives a reasonable description of the data.

In conclusion, at a temperature $T_{em} \approx 70$ K we have observed a change in the temperature dependence of the phase line measured by μSR. The value of $T_{em}$, and also the form of the phase line, both above and below $T_{em}$, are in excellent accord with a recent analysis of the low-field phase diagram.13 This suggests that this phase line traces the decoupling of the vortex lattice, which in the light of complementary SANS measurements seems to occur at the melting transition. We have shown that below $T_{em}$ the electromagnetic coupling dominates over the Josephson coupling of pancake vortices, while above $T_{em}$, where Josephson coupling begins to play a significant role, a parameter free fit is obtained. Furthermore the theory is able to predict the detailed temperature dependence of the μSR linewidth without any additional parameters.

17 SANS data (unpublished).
18 D. Zech (private communication).