Angular dependence of the disorder crossover in the vortex lattice of Bi$_{2.15}$Sr$_{1.85}$CaCu$_2$O$_{8+\delta}$ by muon spin rotation and torque magnetometry

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(Received 11 July 1997)

Using the techniques of muon spin rotation and torque magnetometry, we investigate the crossover field $B_{c_{\alpha}}$ in Bi$_{2.15}$Sr$_{1.85}$CaCu$_2$O$_{8+\delta}$ at which the vortex lattice becomes disordered along the field direction. It is found that $B_{c_{\alpha}}$ scales as the projection of the applied field along the perpendicular to the superconducting planes. This has the implication that a field large enough to give a disordered lattice when applied perpendicular to the planes, can give a well-ordered vortex-line lattice for angles of the field to the $c$ axis greater than a critical value. [S0163-1829(98)09401-6]

I. INTRODUCTION

Cuprate superconductors exhibit a strongly anisotropic superconducting behavior. This is due largely to their layered structure, which favors supercurrents flowing in the CuO$_2$ planes. The high-temperature superconductor Bi$_{2.15}$Sr$_{1.85}$CaCu$_2$O$_{8+\delta}$ (BSCCO) is extremely anisotropic, quantified by the anisotropy parameter $\gamma = \lambda_c/\lambda_{ab} \sim 150$, where $\lambda_c, \lambda_{ab}$ are the penetration depths for currents flowing perpendicular and parallel to the superconducting planes. This extreme anisotropy, together with the high operating temperatures of these high-$T_c$ superconductors, leads to very unusual behavior of flux vortices, such as vortex lattice melting.1,2

In highly anisotropic systems such as these, the flux lines are best viewed as a weakly coupled stack of quasi-two-dimensional (2D) “pancake” vortices, each confined to a superconducting plane.3–5 At low fields and temperatures these stacks may resemble conventional vortex lines, but with increasing fields the response of the system to elastic deformations will change. In this continuum limit the elastic constants used to describe the system are highly dispersive. Above a certain field the energetic cost of interlayer deformations of the lattice, on small scales, becomes less than the cost of intralayer deformations within the superconducting planes. In the extreme limit, where this length scale equals the interlayer separation $s$, so one might view this as a “dimensional crossover.” We shall simply refer to this as the crossover field $B_{c_{\alpha}}$. This crossover is closely related to the so-called “fish-tail” or “second-peak” effect in magnetization hysteresis loops. Many authors have ascribed the fishtail solely to pinning effects,6,7 although it has been shown that the value of the magnetic field at which a microscopic change in the vortex structure, as observed at low temperatures with $\mu$SR, coincides exactly with the appearance of the fishtail.10,11 This microscopic change of the flux structure above $B_{c_{\alpha}}$ is probably accentuated, since short segments of the softening vortex lines can adjust more easily to the random pinning potential. A static disordering of the vortex lines along their length thus occurs. We note that many observable features of this crossover occur because of the influence of the point pinning sites on the vortex lines. This is true for magnetization peak effects, $\mu$SR, and small-angle neutron-scattering (SANS) measurements.4,5,10,11,6 For instance, in macroscopic magnetization measurements the fishtail is due to a sudden increase of the magnetization, which arises from a change in response of the vortices to pinning. We note, however, that these changes most likely reflect the underlying changes in the effective rigidity of the vortex system.

For a strongly Josephson coupled system $B_{c_{\alpha}}$ occurs when the vortex separation $a$ equals the width of the Josephson vortex core $\gamma s$, which gives $B_{c_{\alpha}} = B_J \sim \phi_0/(\gamma s)^2$, where $\phi_0$ is the flux quantum. Recent work suggests that in systems of very large anisotropy, such as BSCCO, this expression may need to be modified in the limit $\lambda_{ab} < \gamma s$.5,12,13 This is because at the field where $a$ first exceeds $\gamma s$, the vortices are still rather dilute, so are well correlated along the $c$ direction and little distorted by intraplane interactions. The crossover is therefore pushed to a higher field $B_{c_{\alpha}} = B_J \sim \gamma s / \lambda_{ab}$, which is the field where the vortices start to overlap strongly.

In this paper we present studies of the angular dependence of the crossover field in BSCCO. We combine $\mu$SR mea-
measurements with torque magnetometry, and show that the signatures revealing the crossover field by both techniques are well correlated. The simple scaling relation which is revealed leads to a simple interpretation of the angular dependence.

II. EXPERIMENT

The BSCCO samples used for the μSR experiments consisted of a mosaic of overdoped single crystals \((T_c = 85 \, \text{K})\), grown using a floating-zone technique described elsewhere.\(^4,18\) The plate-like samples, which have their \(c\) axes perpendicular to their largest face, were mounted with these axes perpendicular to the sample holder. The μSR measurements were carried out on beamline πM3 at the Paul Scherrer Institute, Villigen, Switzerland, and the torque measurements on a homebuilt torque magnetometer at University of Zurich using one single crystal taken from the μSR mosaic.

The μSR experiment uses surface muons of momentum \(29 \, \text{MeV/c}\), which are spin polarized with their spins perpendicular to their momentum direction. The sample is subjected to a magnetic field which is directed parallel to the incident muon momentum. The muons decay after an average lifetime of \(2.2 \, \mu\text{s}\), emitting positrons preferentially along their spin direction.\(^15\) The muons arrive randomly at the sample where they rapidly thermalize and then precess in the local magnetic field. Before the muons hit the sample they pass through a muon detector which starts a clock. Positron detectors placed around the sample measure the number of positrons emitted along a given direction as a function of this clock time. Logic gating of the pulses ensures that the only events accepted are those in which only one muon is present in the sample at any given time. By time-binning large numbers of the detected positron events, a time spectrum is built up which reflects the muon precession within the sample. Inside the mixed state of the superconductor the presence of the vortex lattice gives rise to a spatial modulation of the local flux density \(B(r)\). Since the muons are implanted randomly with respect to the vortex lattice, the range of precession frequencies reflects this internal flux distribution. By Fourier transforming the muon time spectra one obtains the μSR line shape, or frequency distribution. This is an excellent measure of the probability distribution of internal field values of the superconductor, \(p(B)\). The μSR line shape is thus intimately related to the spatial distribution \(B(r)\), so can be used to deduce important structural information on the vortex lattice.\(^3,16,5,12\)

For our analysis we require the μSR line shape, which is a Fourier transform of the positron decay spectrum, in order to obtain \(p(B)\). While conventional Fourier methods may be used, we prefer to utilize a maximum entropy method which suppresses noise due to Poisson counting statistics which would always be present in a conventional Fourier analysis technique.\(^17\) This is useful if one wants to perform systematic analysis on moments of \(p(B)\), which can be extremely sensitive to the presence of random noise. Without any constraints, a uniformly flat distribution \(p(B) = \text{const}\) will maximize the entropy

\[
S = \sum p(B_i) \ln(p(B_i)/d_i),
\]

where \(d_i\) represent default values, which in the analysis were assumed to constitute a flat distribution. In order to obtain the desired field distribution, maximizing the entropy has to be constrained by the data. This is done by introducing the usual quantity \(\chi^2 = \Sigma(n_i - y_i)^2/\sigma_i^2\), where \(n_i\) are the time domain data, after correction for the exponential decay of the muon, \(\sigma_i^2\) are the associated experimental errors, and \(y_i\) constitute the Fourier transform of the calculated field distribution. We then obtain the desired field distribution by minimizing the quantity

\[
L = S - \lambda \chi^2,
\]

where \(\lambda\) is a Lagrange multiplier. In this way our maximum entropy algorithm results in the most uniform distribution which is consistent with the data. Some degree of freedom in weighting the \(\chi^2\) argument with respect to the entropy argument can now be gained by artificially increasing \(\chi^2\) through scaling the errors \(\sigma_i\) by a small fraction. One can thus optimize the agreement between the data and the calculated field distribution by increasing the weight of \(\chi^2\). The criteria we use is to choose that scaling factor which is closest to unity, but which will not increase the noise in the distribution such that the moments can no longer be calculated with confidence. That is, we aim to produce a spectrum which is flat outside of the region of interest. Once this has been achieved, the scaling factor must remain constant for the remainder of a systematic analysis. In the following analysis we used a scaling factor of 1.03. While the choice of this parameter can lead to systematic variations in the spectra, these should be compared with variations which may be introduced in conventional Fourier techniques, such as apodization of the time window. In our method the data itself never undergoes Fourier transformation, so we avoid problems associated with transforming experimental noise. The moments of the distribution may thus be easily evaluated. To avoid artefacts due to systematic variations, for a given analysis we ensure that all data sets have similar counting times and hence similar errors. In summary, the maximum entropy technique should be used with care, but correctly used it provides a very powerful method for examining subtle details of the μSR spectra, not easily possible by other techniques.

For the μSR measurements the sample was mounted on an Fe\(_2\)O\(_3\) backing plate, as described in Refs. 4,18. The purpose of this is to reduce the contributions to the μSR spectra from those muons which to do not stop in the sample, but which land outside of the sample or pass through if the samples are thin. Any muons landing in the antiferromagnet Fe\(_2\)O\(_3\) rapidly depolarize, outside of the observable time window. The recorded spectra thus arise only from muons inside the superconductor. By rotating the sample holder, measurements could be performed for many angles of the applied field to the crystallographic \(c\) direction. This allowed the angular dependence of the vortex behavior to be determined.

The magnetization data was gathered using torque magnetometry. The BSCCO crystals were measured using a cantilever sensor constructed from phosphor bronze (see Ref. 19). A magnetic moment \(\mathbf{m}\) in a field \(\mathbf{B}\) experiences a torque

\[
\tau = \mathbf{m} \times \mathbf{B}.
\]

In our torque magnetometer this torque is mea-
The field distribution in the mixed state of type-II superconductors has been calculated, e.g., by Sidorenko et al.\cite{Sidorenko} For a lattice of straight vortex lines, the field distribution shows a distinct weighting towards fields higher than the average. This is due to the very high fields present in the vortex cores. This is illustrated in Fig. 1(a), which shows a measured line shape for BSCCO with a field of 10 mT directed along the c direction. The expected distribution above the crossover field is quite different, as disorder along the a-b planes, \( m \), is always directed along the c axis, so \( m \) can readily be calculated from the measured torque.

III. RESULTS AND DISCUSSION

The skewness of the field distribution by a quantity derived from the ratio of the third and second moments

\[
\alpha = \frac{\langle (\Delta B)^3 \rangle}{\sqrt[3]{\langle (\Delta B)^2 \rangle}}. \tag{3}
\]

A positive value of \( \alpha \) indicates a weighting of the distribution towards high fields. A symmetric distribution will be quantified by \( \alpha = 0 \). For an ideal lattice of vortex lines, \( \alpha \) is expected from numerical simulations to be \( \sim 1.2 \) for our experimental setup. In Fig. 2 we show the field dependence of \( \alpha \) for different angles. The crossover field is manifested by a drop in \( \alpha \) from its initial value of \( \sim 1.2 \) to \( \sim 0.6 \), indicating a much more symmetric line shape. It can be seen in the figure, that the crossover takes place at higher fields and is also smeared over a wider range of fields if the applied field is at a large angle to the c axis. We determine the crossover field from the point where \( \alpha \) becomes approximately independent of the applied field. This occurs at a lower value of \( \alpha \sim 0.6 \), where we have applied the same criterion previously used to determine the crossover field parallel to the c axis.\cite{note1,note2}

The results of \( B_{cr}(\theta) \) are summarized in Fig. 3. We have also determined \( M(H) \) loops at various different angles of the field with respect to the c axis. As is well known in the literature, these \( M(H) \) loops exhibit a characteristic structure showing two peaks, as shown in Fig. 4(a). In Fig. 4(b) it can be seen that the microscopic change in the field distribution observed by \( \mu \)SR takes place at the deviation of the magnetization from the usual type-II behavior, that is, around the second peak in \( M(H) \).\cite{note3,note4} To agree with the observed crossover in the \( \mu \)SR data, values of the crossover field in Fig. 3 from torque magnetometry are determined from the point of maximum slope of the magnetization at the second peak, which may be seen from the differential of the magnetization also plotted in Fig. 4(b). We see from Fig. 3 that the angular dependence of the crossover field as determined by the two methods of \( \mu \)SR and torque magnetometry both agree, and are well described by the relation

\[
B_{cr}(\theta) = B_{cr}(0)/\cos(\theta). \tag{4}
\]

The crossover field thus effectively depends upon the projection of the applied field along the c direction. This appears to be in accord with the scaling approach of Blatter et al.\cite{Blatter} for anisotropic superconductors in the presence of isotropic disorder. This would predict an angular scaling

\[
\frac{B_{cr}(\theta)}{B_{cr}(0)} \sim \frac{\Delta B_{cr}(\theta)}{\Delta B_{cr}(0)} \sim \Delta B_{cr}(\theta)/\Delta B_{cr}(0) \sim \sqrt[3]{\langle (\Delta B)^2 \rangle}/\sqrt[3]{\langle (\Delta B)^2 \rangle} = \alpha.
\]

FIG. 1. Probability distributions of internal flux density \( P(B) \) in BSCCO at 5 K, as determined by \( \mu \)SR, for fields applied parallel to the c direction. (a) The \( p(B) \) for an applied field of 10 mT, showing a typical line shape for a vortex line lattice. The tail towards high fields arises from fields close to the vortex cores (see text). (b) A typical line shape for fields above the disorder crossover \( B_{cr} \). The truncation of the high-field tail arises from a wandering of the vortex line along the field direction (see text).
With reference to Fig. 2, we note that the apparent rolloff of \( \alpha \) at around 25 mT is remarkably angle independent. In Ref. 12 it was shown that around 20 mT there exists a change in the temperature dependence of the vortex lattice melting line, as measured by \( \muSR \). This was found to be highly consistent with the predictions of Ref. 22, which considers the role of electromagnetic coupling in determining the phase diagram of BSCCO. This position of this feature in the melting line has been interpreted as \( B^* \sim \Phi_0 / (\gamma s)^2 \), the field where the cores of the Josephson vortices start to overlap.\(^{12,22}\) In Ref. 5 the onset of the angle-independent rolloff in \( \alpha \) as a function of field was also associated with \( B^* \). This may be understood as the field above which Josephson currents become ineffective at screening the phase fluctuations arising from pancake vortex displacements in neighboring layers. Electromagnetic coupling then begins to dominate the single vortex line tension, which could lead to a “softening” of the vortex lines at shorter length scales.

Another manifestation of the angular dependence of the crossover field arises from the angular dependence of the second moment of the \( \muSR \) line shape. For a lattice of straight vortex lines, the second moment is expected to scale with the angle of the field with respect to the \( c \) axis as \(^{18,20,23}\)

\[
\langle B^2 \rangle(\vartheta) = \frac{\langle B^2 \rangle(0)}{\left(\cos^2\vartheta + 1/\gamma^2\sin^2\vartheta\right)^{1/2}}.
\]

which for large \( \gamma \) reduces to Eq. (4). This agreement is, however, rather surprising, given that the crossover occurs at low field, outside the region of validity of the Blatter scaling relation.

\[
\langle \Delta B^2 \rangle^{1/2}(\vartheta) = \langle \Delta B^2 \rangle^{1/2}(0)\left(\cos^2\vartheta + 1/\gamma^2\sin^2\vartheta\right)^{1/2},
\]

which for compounds as anisotropic as BSCCO reduces to

\[
\langle \Delta B^2 \rangle^{1/2}(\vartheta) = \langle \Delta B^2 \rangle^{1/2}(0)\cos(\vartheta).
\]

For fields below the crossover field we observe a similar angular dependence in BSCCO as is evident from Fig. 5. For fields well above the crossover field, however, a linewidth only weakly dependent on the angle of the applied field is observed (cf. Fig. 5, Ref. 18). This can be understood in terms of the pinning-induced disorder of the vortex lattice above \( B^* \), as described above. Random lateral displacements of vortices in neighboring layers, due to pinning will induce phase fluctuations of the order parameter. This will also lead to a random array of Josephson vortices, generating fields parallel to the planes. There is thus a large variation in the local field distribution arising from all components of the internal field. Hence, disorder continues to dominate the field distribution, even when the field is tilted away from the \( c \) axis. As a result, the second moment remains largely independent of angle, even for quite
large values of $\theta$. An interesting phenomenon occurs if the angle $\theta$ is such that the component of $B$ along the $c$ axis is smaller than the crossover field, $B \cos(\theta) < B_{c}(0)$. In this case the pancake vortices appear to move towards being well aligned along the field direction, which is manifest in two ways. Firstly, the asymmetry parameter $\alpha$ rises towards its ideal lattice value, as can be seen by the clear change of line shape at 90 mT, above and below $\sim 40^\circ$, in Figs. 5 and 6. Secondly, the angular variation of the second moment rejoins the angular dependence which would be expected for a well ordered lattice, as depicted in Fig. 5. We interpret this as follows. The scaling of Eq. (5) implies that as the angle increases above some critical angle $\theta_c$, the applied field $B$ will be less than the angular-dependent crossover field, that is $B < B_{c}(\theta_c)$. For high angles the lattice thus becomes more ordered. Taking $B = 90$ mT yields $\theta_c \sim 41^\circ$, consistent with the observed change of behavior at $\sim 40^\circ$. Conversely, we can thus estimate the value of the crossover field at certain angles from such changes of behavior of the angular dependence of the $\mu$SR linewidth. Some estimates of the crossover using this method are also included in Fig. 3.

In conclusion, we have studied the disorder crossover field in BISCOO using the techniques of $\mu$SR and torque magnetometry. We find that the crossover field scales with the field component in the $c$ direction. This result is obtained from both a microscopic change in the field distribution as shown by $\mu$SR, and from the deviation of $M(H)$ loops from the usual type-II behavior in the so-called fishtail. We have also studied the angular dependence of the width of the field distribution in $\mu$SR for different applied fields. We find the expected angular dependence for fields below the crossover, whereas it is only weakly dependent on the angle for very high fields. In an intermediate regime however, where the applied field is $B \approx B_{c}(0)$, we find a more complex behavior, observing a crossover of the lattice from disordered to quasiordered with increasing angle. From this we are also able to determine the angular dependence of the crossover field, which is in good agreement with that determined from the skewness of the field distribution.

**ACKNOWLEDGMENTS**

We would like to thank T. W. Li (University of Leiden) for the preparation of the crystals, and the staff on beamline m3 at PSI, Zurich, for technical support. Financial support from the Swiss National Science Foundation and the EPSRC of the UK is gratefully acknowledged.

FIG. 6. The data are all field-cooled measurements at 5 K. At high fields $\sim 5$ K, the crossover happens ($\sim 0^\circ$ for 90 mT), we have another determination of the angular dependence of the crossover field. (b) The line-shape asymmetry parameter $\alpha$ [see Eq. (3)] as a function of angle as above. At low fields (e.g., 40 mT) the value of $\alpha$ is as expected for an ideal vortex line lattice (see text) for all angles. At high fields (e.g., 300 mT) $\alpha$ is again independent of the angle, but has a value reflecting the disorder present above the crossover. For intermediate fields (e.g., 90 mT) the rise in $\alpha$ above some critical angle reflects the increasing order in the vortex lattice as $B$ effectively falls below the crossover field (see text). This change in $\alpha$ is a measure of the change in the line-shapes seen in Fig. 6. The data are all field-cooled measurements at 5 K.
13 V. M. Vinokur (private communication).