Combining approximate zero constraints for measurement invariance and cross-loadings: An application of dual process growth curve models with panel data

Daniel Seddig

University of Zurich UZH

Insitute of Sociology

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$$\boldsymbol{Y}_{k,t} = \boldsymbol{T}_{k,t} + \Lambda_{k,t}\eta_t + \Theta_{\varepsilon_{k,t}} \qquad k = 1,\ldots,K; t = 1,\ldots,T \quad (1)$$

$$\boldsymbol{Y}_{k,t} = \boldsymbol{T}_{k,t} + \Lambda_{k,t}\eta_t + \Theta_{\varepsilon_{k,t}} \qquad k = 1,\ldots,K; t = 1,\ldots,T \quad (1)$$

(a) Exact MI in $\Lambda_{k,t}$:

$$\begin{array}{ccccccc} \lambda_{1,1} & = & \lambda_{1,2} & = & \ldots & = & \lambda_{1,T} \\ \lambda_{2,1} & = & \lambda_{2,2} & = & \ldots & = & \lambda_{2,T} \\ \vdots & = & \vdots & = & \ldots & = & \vdots \\ \lambda_{K,1} & = & \lambda_{K,2} & = & \ldots & = & \lambda_{K,T} \end{array} \quad (2)$$

- Highest level of stringency
- Differences across group/time exactly zero

$$\boldsymbol{Y}_{k,t} = \boldsymbol{T}_{k,t} + \Lambda_{k,t}\eta_t + \Theta_{\varepsilon_{k,t}} \qquad k = 1,\ldots,K; t = 1,\ldots,T \tag{1}$$

(a) Exact MI in $\Lambda_{k,t}$:

$$\begin{matrix} \lambda_{1,1} & = & \lambda_{1,2} & = & \ldots & = & \lambda_{1,T} \\ \lambda_{2,1} & = & \lambda_{2,2} & = & \ldots & = & \lambda_{2,T} \\ \vdots & = & \vdots & = & \ldots & = & \vdots \\ \lambda_{K,1} & = & \lambda_{K,2} & = & \ldots & = & \lambda_{K,T} \end{matrix} \tag{2}$$

- Highest level of stringency
- Differences across group/time exactly zero

(b) Approximate MI in $\Lambda_{k,t}$:

$$\begin{matrix} \lambda_{1,1} & \approx & \lambda_{1,2} & \approx & \ldots & \approx & \lambda_{1,T} \\ \lambda_{2,1} & \approx & \lambda_{2,2} & \approx & \ldots & \approx & \lambda_{2,T} \\ \vdots & \approx & \vdots & \approx & \ldots & \approx & \vdots \\ \lambda_{K,1} & \approx & \lambda_{K,2} & \approx & \ldots & \approx & \lambda_{K,T} \end{matrix} \tag{3}$$

- flexibility, "wiggle room" (Van de Schoot et al., 2013)
- identification of non-invariants: "two-step Bayesian analysis procedure" (Muthén & Asparouhov, 2013)

Prior distributions for approximate zero constraints

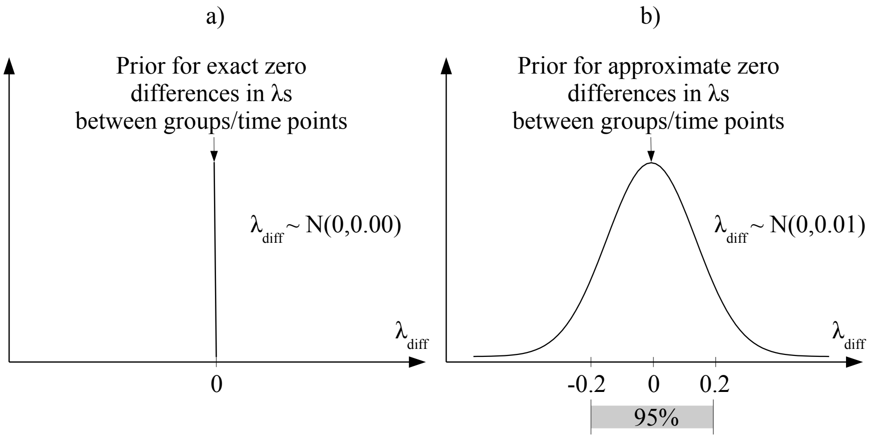


Figure 1: Priors for exact (a) and approximate (b) MI

Bayesian SEM
Cross-loadings

- Exact zero cross-loadings

$$\begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \\ y_6 \\ y_7 \\ y_8 \end{pmatrix} = \begin{pmatrix} \lambda_{y_{11}} = 1 & \lambda_{y_{12}} = 0 \\ \lambda_{y_{21}} & \lambda_{y_{22}} = 0 \\ \lambda_{y_{31}} & \lambda_{y_{32}} = 0 \\ \lambda_{y_{41}} & \lambda_{y_{42}} = 0 \\ \lambda_{y_{51}} = 0 & \lambda_{y_{52}} = 1 \\ \lambda_{y_{61}} = 0 & \lambda_{y_{62}} \\ \lambda_{y_{71}} = 0 & \lambda_{y_{72}} \\ \lambda_{y_{81}} = 0 & \lambda_{y_{82}} \end{pmatrix} * \begin{pmatrix} \eta_1 \\ \eta_2 \end{pmatrix} + \begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \varepsilon_4 \\ \varepsilon_5 \\ \varepsilon_6 \\ \varepsilon_7 \\ \varepsilon_8 \end{pmatrix} \tag{4}$$

- Exact zero cross-loadings

$$\begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \\ y_6 \\ y_7 \\ y_8 \end{pmatrix} = \begin{pmatrix} \lambda_{y_{11}} = 1 & \lambda_{y_{12}} = 0 \\ \lambda_{y_{21}} & \lambda_{y_{22}} = 0 \\ \lambda_{y_{31}} & \lambda_{y_{32}} = 0 \\ \lambda_{y_{41}} & \lambda_{y_{42}} = 0 \\ \lambda_{y_{51}} = 0 & \lambda_{y_{52}} = 1 \\ \lambda_{y_{61}} = 0 & \lambda_{y_{62}} \\ \lambda_{y_{71}} = 0 & \lambda_{y_{72}} \\ \lambda_{y_{81}} = 0 & \lambda_{y_{82}} \end{pmatrix} * \begin{pmatrix} \eta_1 \\ \eta_2 \end{pmatrix} + \begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \varepsilon_4 \\ \varepsilon_5 \\ \varepsilon_6 \\ \varepsilon_7 \\ \varepsilon_8 \end{pmatrix} \tag{4}$$

- Approximate zero cross-loadings

$$\begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \\ y_6 \\ y_7 \\ y_8 \end{pmatrix} = \begin{pmatrix} \lambda_{y_{11}} = 1 & \lambda_{y_{12}} \approx 0 \\ \lambda_{y_{21}} & \lambda_{y_{22}} \approx 0 \\ \lambda_{y_{31}} & \lambda_{y_{32}} \approx 0 \\ \lambda_{y_{41}} & \lambda_{y_{42}} \approx 0 \\ \lambda_{y_{51}} \approx 0 & \lambda_{y_{52}} = 1 \\ \lambda_{y_{61}} \approx 0 & \lambda_{y_{62}} \\ \lambda_{y_{71}} \approx 0 & \lambda_{y_{72}} \\ \lambda_{y_{81}} \approx 0 & \lambda_{y_{82}} \end{pmatrix} * \begin{pmatrix} \eta_1 \\ \eta_2 \end{pmatrix} + \begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \varepsilon_4 \\ \varepsilon_5 \\ \varepsilon_6 \\ \varepsilon_7 \\ \varepsilon_8 \end{pmatrix} \tag{5}$$

Illustration

Hedonism and associating with delinquent peer groups in adolescence

Illustration

Hedonism and associating with delinquent peer groups in adolescence

- Hedonism: Pleasure and sensuous gratification
- Stimulation: Excitement, novelty, and challenge in life
- Hedonism/Stimulation $\leftrightarrow$ Delinquent Peer Groups
- Development: as adolescents interest in both dimensions decreases, associations with delinquent peer groups decreases

1. Bayesian SEM
   - Measurement invariance
   - Cross-loadings

2. Illustration
   - Substantive question
   - Data
   - Results I: cross-loadings
   - Results II: measurement invariance
   - Results III: final model

- "Crimoc-study"; German criminological panel study
- Panel data; n=357 male respondents; ages 14 to 20
- Beliefs about hedonism/stimulation (scaled 1-5):
  1. $h1$: understanding for people who do what they desire
  2. $h2$: need for excitement
  3. $h3$: living a life of pleasure
- Association with violent peer group (scaled 1-5):
  1. $g1$: group enforces interests with force
  2. $g2$: group involved in brawls

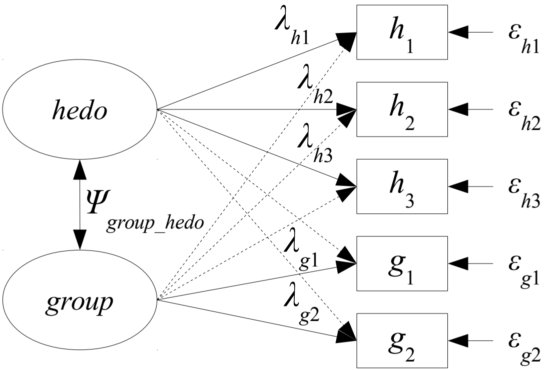


Figure 2: CFA with cross-loadings

Table 1: BCFA model assessment for age 18 (n=357)

Prior (λ_{CL})	BIC	DIC	PPP
∼N(0,0.000)	5021	4958	0.030
∼N(0,0.001)	5045	4953	0.102
∼N(0,0.010)	5033	4944	0.453
∼N(0,0.050)	5031	4943	0.509
∼N(0,0.100)	5031	4943	0.512

Note: BIC = Bayesian information criterion; DIC = deviance information criterion; PPP = posterior predictive p-value.

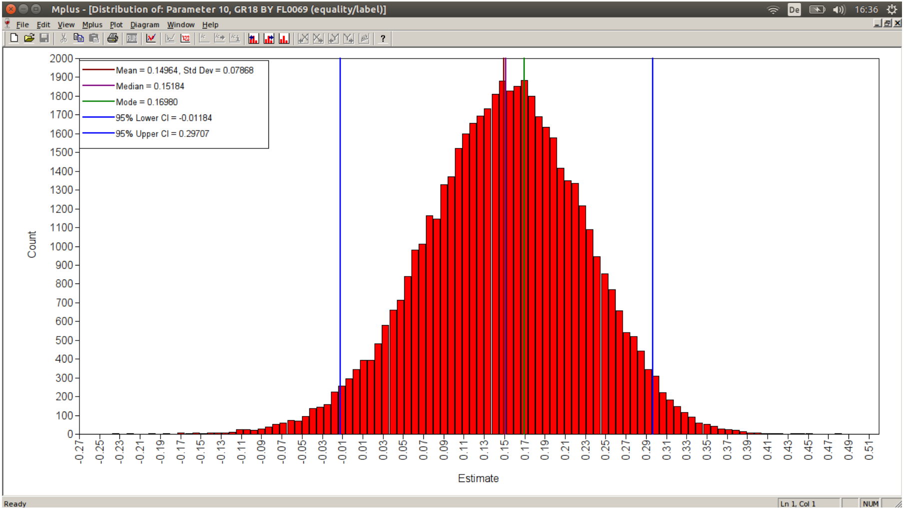
Illustration

Cross-loadings: BCFA with ∼N (0,0.010)

	Estimate	Posterior S.D.	One-Tailed P-Value	95% C.I. Lower 2.5%	Upper 2.5%
group_18 BY					
g1_18	1.000	0.000	0.000	1.000	1.000
g2_18	0.856	0.159	0.000	0.652	1.242
h1_18	-0.063	0.087	0.233	-0.235	0.104
h2_18	0.152	0.079	0.034	-0.012	0.297
h3_18	-0.012	0.076	0.438	-0.167	0.133
hedo_18 BY					
h1_18	1.000	0.000	0.000	1.000	1.000
h2_18	0.563	0.168	0.000	0.293	0.951
h3_18	0.572	0.153	0.000	0.316	0.919
g1_18	0.002	0.083	0.492	-0.158	0.168
g2_18	0.004	0.077	0.477	-0.156	0.151
STDYX Standardization					
hedo_18 WITH					
group_18	0.404	0.118	0.002	0.145	0.605

Illustration

Cross-loadings: Posterior distribution of cross-loading for item "h2_18"



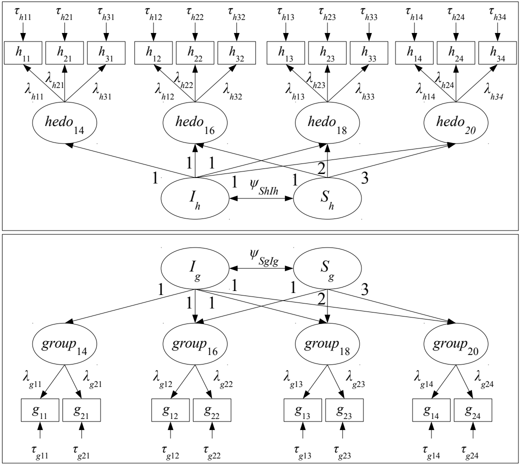


Figure 3: LGMs for *hedo* and *group*

Table 2: Univariate LGM assessment with scalar MI (n=357)

			hedo			*group*	
Mode	Prior (λ_{diff})	BIC	DIC	PPP	BIC	DIC	PPP
Exact	∼N(0,0.000)	12262	12092	0.010	6057	5934	0.381
Appr.	∼N(0,0.001)	12337	12073	0.183	6121	5933	0.458
	∼N(0,0.010)	12331	12072	0.252	6117	5933	0.543
	∼N(0,0.050)	12329	12070	0.263	6116	5930	0.545
Partial	∼N(0,0.000)	12251	12078	0.071	-	-	-

Note: BIC = Bayesian information criterion; DIC = deviance information criterion; PPP = posterior predictive p-value.

Illustration

Hedonism: univariate LGM factor loadings with ~N(0,0.010)

	Estimate	Posterior S.D.	One-Tailed P-Value	95% C.I. Lower 2.5%	95% C.I. Upper 2.5%
hedo_14 BY					
h1_14	1.000	0.000	0.000	1.000	1.000
h2_14	0.694	0.089	0.000	0.531	0.882
h3_14	0.759	0.099	0.000	0.578	0.969
hedo_16 BY					
h1_16	0.970	0.042	0.000	0.890	1.054
h2_16	0.745	0.091	0.000	0.578	0.935
h3_16	0.789	0.100	0.000	0.608	1.001
hedo_18 BY					
h1_18	0.953	0.052	0.000	0.855	1.057
h2_18	0.709	0.096	0.000	0.535	0.912
h3_18	0.788	0.106	0.000	0.596	1.012
hedo_20 BY					
h1_20	0.922	0.065	0.000	0.801	1.056
h2_20	0.702	0.101	0.000	0.517	0.917
h3_20	0.829	0.112	0.000	0.625	1.066

Illustration

Hedonism: univariate LGM factor loadings with ∼N(0,0.050)

	Estimate	Posterior S.D.	One-Tailed P-Value	95% C.I. Lower 2.5%	95% C.I. Upper 2.5%
hedo_14 BY					
h1_14	1.000	0.000	0.000	1.000	1.000
h2_14	0.706	0.097	0.000	0.530	0.910
h3_14	0.773	0.107	0.000	0.581	0.999
hedo_16 BY					
h1_16	0.939	0.082	0.000	0.777	1.101
h2_16	0.746	0.102	0.000	0.560	0.960
h3_16	0.780	0.110	0.000	0.581	1.014
hedo_18 BY					
h1_18	0.962	0.110	0.000	0.742	1.173
h2_18	0.680	0.119	0.000	0.465	0.929
h3_18	0.770	0.133	0.000	0.529	1.049
hedo_20 BY					
h1_20	0.868	0.138	0.000	0.602	1.138
h2_20	0.663	0.137	0.000	0.416	0.954
h3_20	0.798	0.156	0.000	0.511	1.119

Illustration

Multivariate LGM

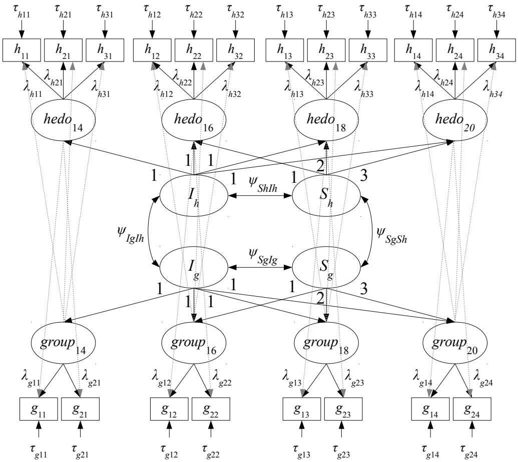


Figure 4: Multivariate LGM with cross-loadings

Table 3: Multivariate LGM assessment with scalar MI (n=357)

Mode	Prior (λ_{diff} / τ_{diff})	Prior (CLs)	BIC	DIC	PPP
Exact MI w/o CLs	$\sim$N(0,0.000)	$\sim$N(0,0.000)	18232	17971	0.000
Exact MI w/ CLs	$\sim$N(0,0.000)	$\sim$N(0,0.010)	18267	17918	0.171
Appr. MI w/o CLs	$\sim$N(0,0.010)	$\sim$N(0,0.000)	18414	17954	0.025
Appr. MI w/ CLs	$\sim$N(0,0.010)	$\sim$N(0,0.010)	18479	17919	0.372

Note: BIC = Bayesian information criterion; DIC = deviance information criterion; PPP = posterior predictive p-value.

	Estimate	Posterior S.D.	One-Tailed P-Value	95% C.I. Lower 2.5%	95% C.I. Upper 2.5%
Means					
I_HE	2.967	0.124	0.000	2.724	3.211
S_HE	-0.118	0.073	0.059	-0.253	0.032
I_GR	1.541	0.190	0.000	1.173	1.917
S_GR	-0.091	0.094	0.164	-0.275	0.100
STDYX Standardization					
I_HE WITH					
I_GR	0.580	0.142	0.000	0.284	0.833
S_HE WITH					
S_GR	0.463	0.194	0.010	0.071	0.828

```
                            Posterior  One-Tailed         95% C.I.
              Estimate        S.D.      P-Value    Lower 2.5%  Upper 2.5%
   Means
     I_HE        2.967       0.124       0.000       2.724       3.211
     S_HE       -0.118       0.073       0.059      -0.253       0.032
     I_GR        1.541       0.190       0.000       1.173       1.917
     S_GR       -0.091       0.094       0.164      -0.275       0.100

 STDYX Standardization
   I_HE WITH
     I_GR        0.580       0.142       0.000       0.284       0.833
        (ML = 0.778)
   S_HE WITH
     S_GR        0.463       0.194       0.010       0.071       0.828
        (ML = 0.680)
```

Summary

- BSEM useful
- but...
  - Prior choice may be an obstacle
  - Compromise between fit and precision
  - Giving up parsimony vs. using prior assumptions

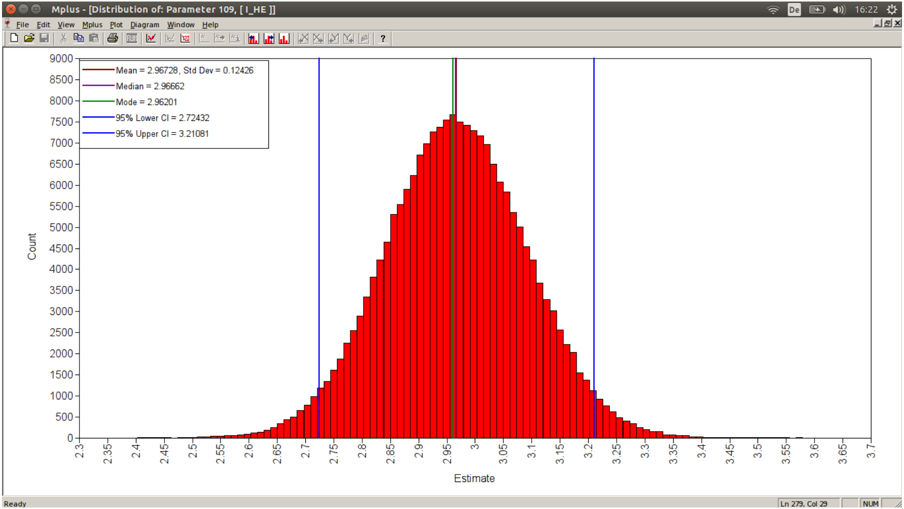
Thank you!

University of Zurich[UZH]

Daniel Seddig
Institute of Sociology
seddig@soziologie.uzh.ch

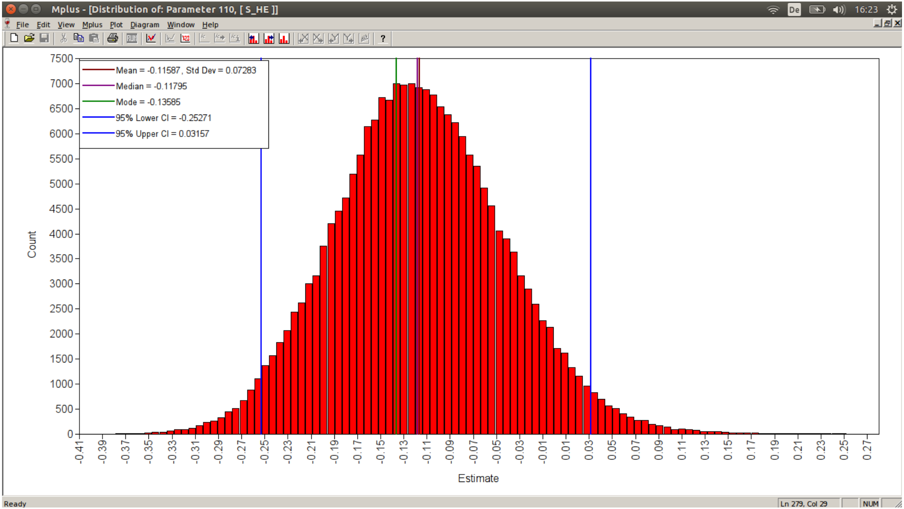
Appendix

Multivariate LGM: Posterior distribution of intercept mean (hedonism)



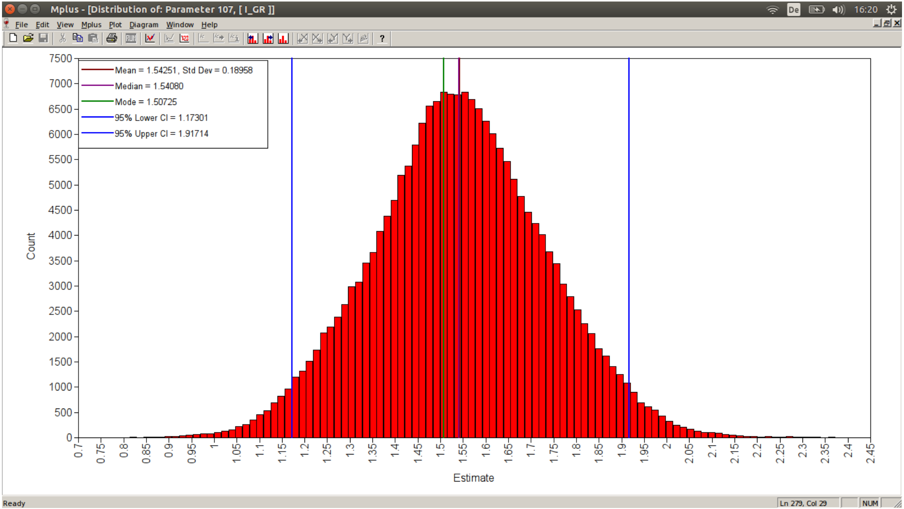
Appendix

Multivariate LGM: Posterior distribution of slope mean (hedonism)



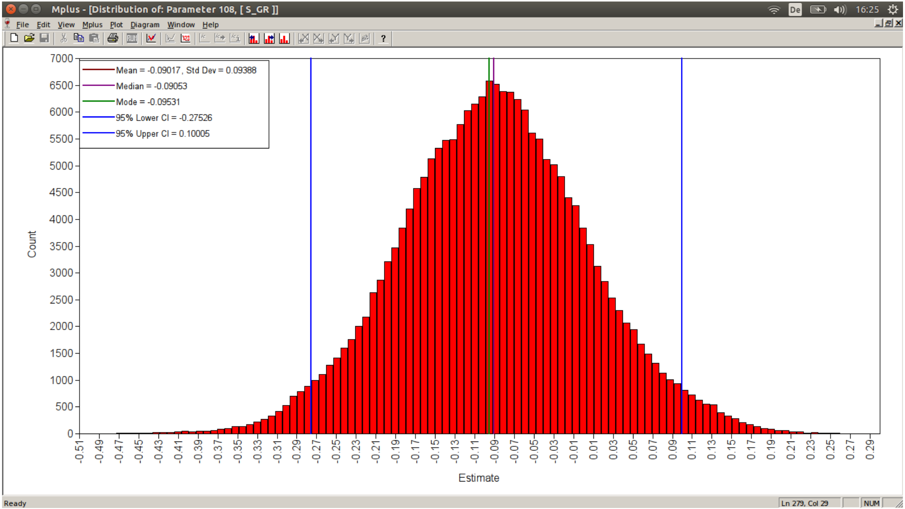
Appendix

Multivariate LGM: Posterior distribution of intercept mean (peer group)



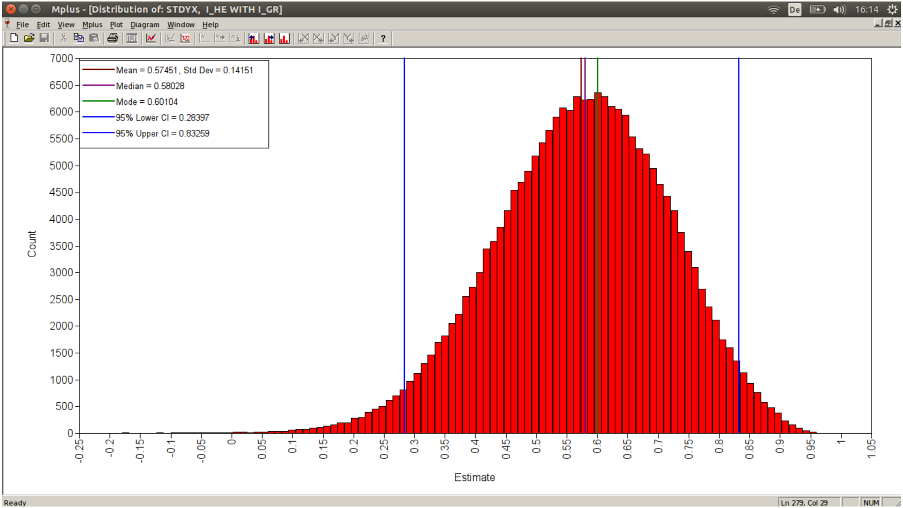
Appendix

Multivariate LGM: Posterior distribution of slope mean (group)



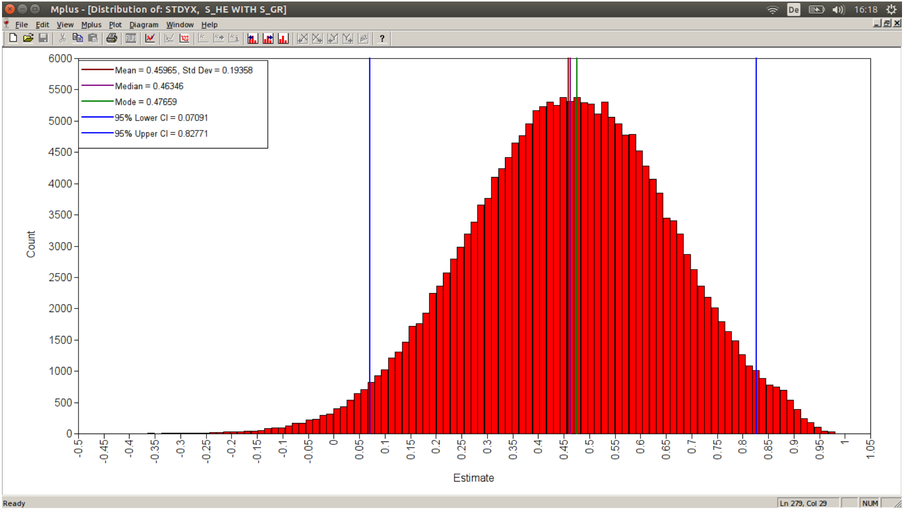
Appendix

Multivariate LGM: Posterior distribution of intercept correlation



Appendix

Multivariate LGM: Posterior distribution of slope correlation



```
Analysis:
  Estimator=Bayes;
  Chains=2;
  Proc=2;
  Biterations=1000000(200000);
  Bseed=3010;
```

Appendix
Convergence

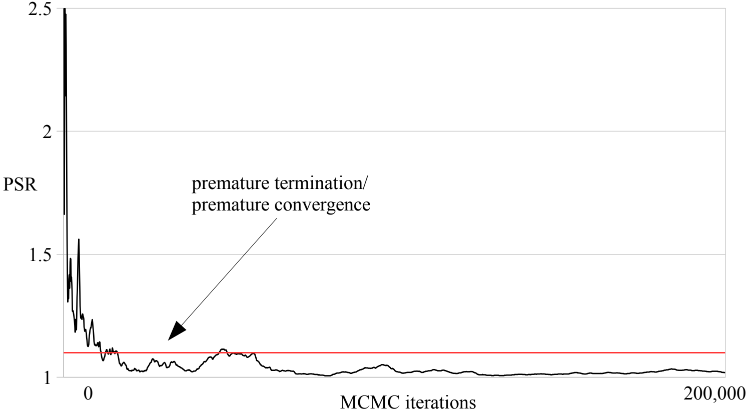


Figure 5: Potential scale reduction factor (PSR) plot

Appendix

M*plus* Input: approximate MI

```
Model:
  gr14 by bc0054@1 bc0056 (a12) !marker item "g1" lambda
  bl0054 bl0069 bl0076 (cl1-cl3); !cross-loadings
  [bc0054@0]; !marker item "g1" tau
  [bc0056] (b12);

  gr16 by dc0054* dc0056* (a21-a22)
  dl0054* dl0069* dl0076* (cl4-cl6); !cross-loadings
  [dc0054 dc0056] (b21-b22);

  gr18 by fc0054* fc0056* (a31-a32)
  fl0054* fl0069* fl0076* (cl7-cl9); !cross-loadings
  [fc0054 fc0056] (b31-b32);

  gr20 by hc0054* hc0056* (a41-a42)
  hl0054* hl0069* hl0076* (cl10-cl12); !cross-loadings
  [hc0054 hc0056] (b41-b42);
```

Appendix

M*plus* Input: approximate MI

```
he14 by bl0054@1 bl0069 bl0076 (c12-c13)
bc0054 bc0056 (cl13-cl14);
[bl0054@0];
[bl0069 bl0076] (d12-d13);

he16 by dl0054* dl0069* dl0076* (c21-c23)
dc0054* dc0056* (cl15-cl16);
[dl0054 dl0069 dl0076] (d21-d23);

he18 by fl0054* fl0069* fl0076* (c31-c33)
fc0054* fc0056* (cl17-cl18);
[fl0054 fl0069 fl0076] (d31-d33);

he20 by hl0054* hl0069* hl0076* (c41-c43)
hc0054* hc0056* (cl19-cl20);
[hl0054 hl0069 hl0076] (d41-d43);

i_gr s_gr | gr14@0 gr16@1 gr18@2 gr20@3;
[i_gr s_gr];
i_he s_he | he14@0 he16@1 he18@2 he20@3;
[i_he s_he];
```

Mplus Input: approximate MI

```
Model priors:
  Do(2,2) diff(a1#-a4#)~N(0,0.01); !"do diff" for lambda -differences
  Do(2,2) diff(b1#-b4#)~N(0,0.01); !"do diff" for tau-differences

  a21~N(1,0.01); !priors for marker item "g1" lambda
  a31~N(1,0.01);
  a41~N(1,0.01);

  b21~N(0,0.01); !priors for marker item "g1" tau
  b31~N(0,0.01);
  b41~N(0,0.01);

  Do(2,3) diff(c1#-c4#)~N(0,0.01);
  Do(2,3) diff(d1#-d4#)~N(0,0.01);

  c21~N(1,0.01);
  c31~N(1,0.01);
  c41~N(1,0.01);

  d21~N(0,0.01);
  d31~N(0,0.01);
  d41~N(0,0.01);
```

```
Model constraint:
  NEW(a11 ave1 diff1_1-diff1_4); !calculation of differences between
  a11=1;                          !marker item lambdas and their
  ave1=(a11+a21+a31+a41)/4;       !average across time points
  Do(1,4) diff1_#=a#1-ave1;

  NEW(b11 ave2 diff2_1-diff2_4); !calculation of differences between
  b11=0;                          !marker item taus and their
  ave2=(b11+b21+b31+b41)/4;       !average across time points
  Do(1,4) diff2_#=b#1-ave2;

  NEW(c11 ave3 diff3_1-diff3_4);
  c11=1;
  ave3=(c11+c21+c31+c41)/4;
  Do(1,4) diff3_#=c#1-ave3;

  NEW(d11 ave4 diff4_1-diff4_4);
  d11=0;
  ave4=(d11+d21+d31+d41)/4;
  Do(1,4) diff4_#=d#1-ave4;
```