CP-violation in hadronic kaon decays from lattice QCD

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Big question: matter-antimatter asymmetry

The visible Universe is almost entirely made of matter rather than antimatter.
Sakharov conditions:

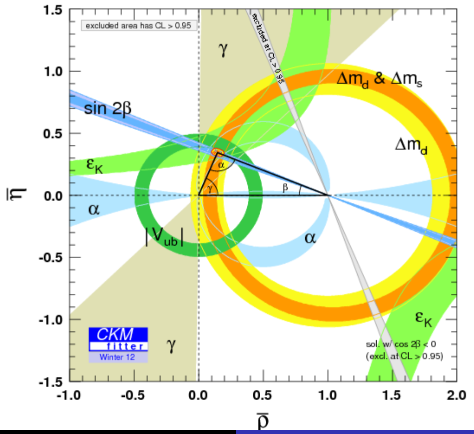
- Out of thermal equilibrium
- Baryon number violation - must have more baryons than antibaryons
- **C and CP violation** - different branching ratios for particles and antiparticles

Practically all CP-violation in the Standard Model comes from the Yukawa coulplings $\rightarrow$ CKM and PMNS matrix elements below electroweak scale.
$V_{ij} W_\mu \bar{q}_i \gamma^\mu (1-\gamma^5) q_j$

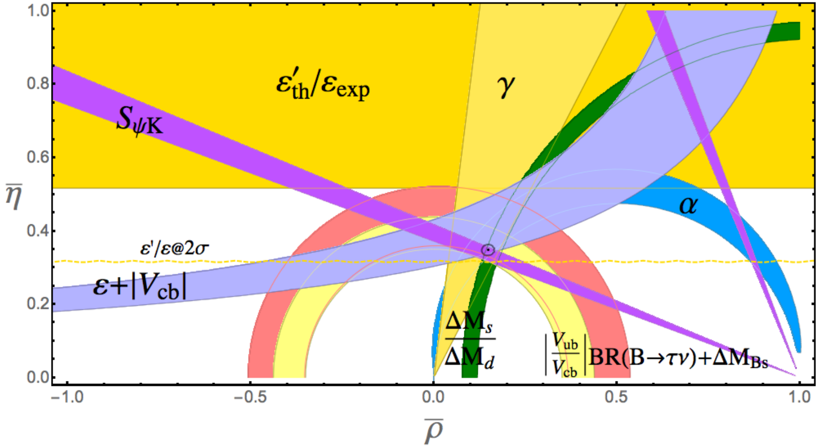
CKM unitarity triangle

In SM the CKM matrix V_{ij} is unitary: $V_{ij}^* V_{ik} = \delta_{jk}$, and hence

$$1 + \frac{V_{us}^* V_{ub}}{V_{cs}^* V_{cb}} + \frac{V_{ts}^* V_{tb}}{V_{cs}^* V_{cb}} = 0 \qquad (1)$$



Additional constraint - ε'/ε



(1508.01801)

CP violation - indirect

Flavour eigenstates:

$$|K^0\rangle = |\bar{s}d\rangle, \quad |\bar{K}^0\rangle = |\bar{d}s\rangle \tag{2}$$

We can then construct CP-eigenstates:

$$|K^0_\pm\rangle = \frac{1}{\sqrt{2}}\left(|K^0\rangle \pm |\bar{K}^0\rangle\right)$$
$$CP\,|K^0_\pm\rangle = \pm\,|K^0_\pm\rangle$$

If CP was conserved these would also be the eigenstates of the Hamiltonian. Otherwise, Hamiltonian eigenstates can be written as:

$$|K_S\rangle = \frac{1}{\sqrt{1+|\bar{\varepsilon}|^2}}\left(|K^0_+\rangle + \bar{\varepsilon}\,|K^0_-\rangle\right)$$
$$|K_L\rangle = \frac{1}{\sqrt{1+|\bar{\varepsilon}|^2}}\left(|K^0_-\rangle + \bar{\varepsilon}\,|K^0_+\rangle\right)$$

CP violation - direct

We can also have CP violation in the matrix element. If CP invariance holds the transfer matrix satisfies:

$$(CP)^{-1} T (CP) = T$$

Consider a system with two possible final states which are CP-eigenstates (e.g. $K \to \pi\pi$)

$$\langle f \mid T \mid i \rangle = e^{i(\xi_i)} \eta_f \langle f \mid T \mid \bar{i} \rangle$$
$$\langle g \mid T \mid i \rangle = e^{i(\xi_i)} \eta_g \langle g \mid T \mid \bar{i} \rangle$$

with $\eta_f, \eta_g = \pm 1$. Hence the quantity

$$\eta_g \langle f \mid T \mid i \rangle \langle g \mid T \mid \bar{i} \rangle - \eta_f \langle f \mid T \mid \bar{i} \rangle \langle g \mid T \mid i \rangle$$

violates CP.

We need at least **two** possible final states to be sensitive to direct CP-violation.

In $K \to \pi\pi$ the $\pi\pi$ states can be distinguished by their *isospin*.

Isospin

$\alpha_s >> \alpha_{em}$, $m_d - m_u << \Lambda_{QCD}$, so we can define an approximate SU(2) symmetry connecting up and down quarks:

Quark	Isospin	I_3
$u/\bar{d}$	1/2	+1/2
$d/\bar{u}$	1/2	-1/2
other	0	0

Pions form an isospin triplet:

Pion	Quark content	I_3
π^+	$\bar{d}u$	+1
π^0	$\frac{1}{\sqrt{2}}\left(\bar{u}u - \bar{d}d\right)$	0
π^-	$\bar{u}d$	-1

Kaons consist of 1 stange quark/antiquark and 1 light antiquark/quark and therefore $I = 1/2$

- I=2

$$|2,2\rangle = |\pi^+\pi^+\rangle$$
$$|2,1\rangle = \frac{1}{\sqrt{2}}\left(|\pi^+\pi^0\rangle + |\pi^0\pi^+\rangle\right)$$
$$|2,0\rangle = \frac{1}{\sqrt{6}}\left(|\pi^+\pi^-\rangle + |\pi^-\pi^+\rangle + 2\,|\pi^0\pi^0\rangle\right)$$

- I=1

$$|1,1\rangle = \frac{1}{\sqrt{2}}\left(|\pi^+\pi^0\rangle - |\pi^0\pi^+\rangle\right)$$
$$|1,0\rangle = \frac{1}{\sqrt{2}}\left(|\pi^+\pi^-\rangle - |\pi^-\pi^+\rangle\right)$$

- I=0

$$|0,0\rangle = \frac{1}{\sqrt{3}}\left(|\pi^+\pi^-\rangle + |\pi^-\pi^+\rangle - |\pi^0\pi^0\rangle\right)$$

Isospin 3

$\pi\pi$ I=1 states are forbidden in $K \to \pi\pi$ decays by the following argument.

- Both kaon and pions are particles with spin $J = 0$
- Angular momentum conservation gives the orbital angular momentum of the two-pion state: $L = 0$ (i.e. 's-wave')
- Under parity the $L = 0$ partial wave is parity *even*
- I=1 state is (repeated):

$$|1,1\rangle = \frac{1}{\sqrt{2}} \left(|\pi^+\pi^0\rangle - |\pi^0\pi^+\rangle \right),$$

which is parity *odd*.

Kaon system - measurable quantities

On this slide only - $|0\rangle$ and $|2\rangle$ are $\pi\pi$ eigenstates with isospin 0 and 2 respectively, T is the transition matrix.

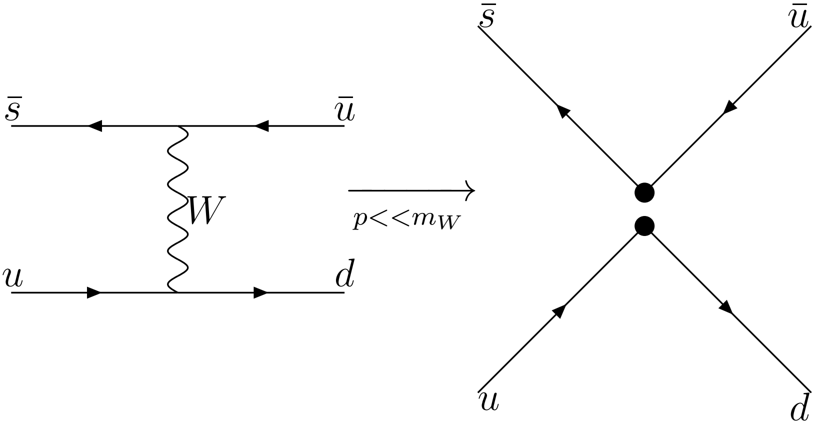
$$\omega \equiv \frac{\langle 2 \mid T \mid K_S\rangle}{\langle 0 \mid T \mid K_S\rangle}$$

$$\varepsilon \equiv \frac{\langle 0 \mid T \mid K_L\rangle}{\langle 0 \mid T \mid K_S\rangle}$$

$$\varepsilon' \equiv \frac{\langle 2 \mid T \mid K_L\rangle\langle 0 \mid T \mid K_S\rangle - \langle 2 \mid T \mid K_S\rangle\langle 0 \mid T \mid K_L\rangle}{\sqrt{2}\langle 0 \mid T \mid K_S\rangle^2}$$

$$\propto \langle 2 \mid T \mid K^0\rangle\langle 0 \mid T \mid \bar{K}^0\rangle - \langle 2 \mid T \mid K^0\rangle\langle 0 \mid T \mid \bar{K}^0\rangle$$

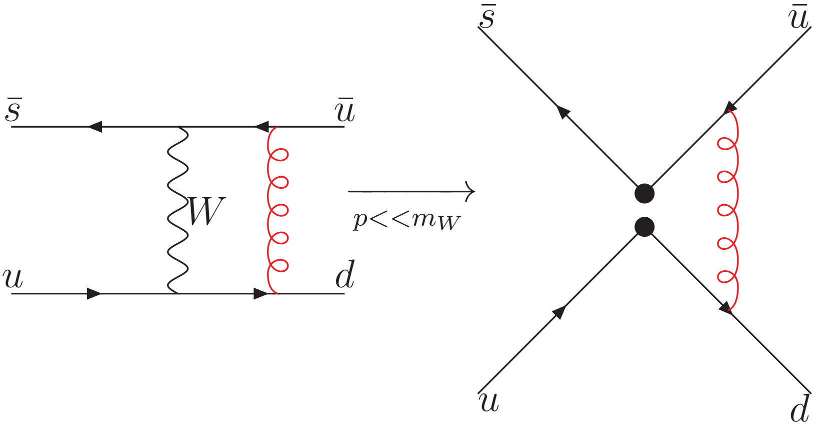
- ε' measures *direct* CP violation - CP violated in the interaction.
- ε measures *indirect* CP violation - in mixing and interference.
- Experimentally $1/|\omega| \approx 22.45$. This is known as '$\Delta I = 1/2$ rule'.
- Also experimantally, $\mathrm{Re}(\varepsilon'/\varepsilon) = 1.65(26) \times 10^{-3}$.



$$\frac{ig^2}{p^2 - m_W^2 + i\varepsilon} \to \frac{G_F}{\sqrt{2}}$$

We will only focus on contributions up to $O(G_F)$, so that $\langle \pi\pi \mid T \mid K \rangle \approx \langle \pi\pi \mid H_W \mid K \rangle$

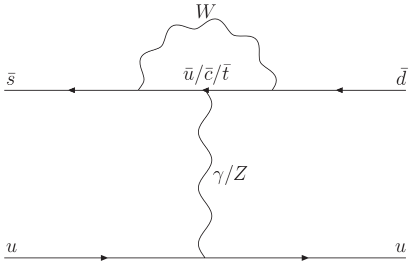
Operator product expansion 2



$$\langle H_W \rangle = V_{us}^* V_{ud} \frac{G_F}{\sqrt{2}} \sum_i C_i(\mu) \langle \mathcal{O}_i(\mu) \rangle$$

Penguin diagrams

Each penguin diagram contributes as:



$$c_1 V_{us}^* V_{ud} + c_2 \underbrace{V_{cs}^* V_{cd}}_{-V_{us}^* V_{ud} - V_{ts}^* V_{td}} + c_3 V_{ts}^* V_{td} = V_{us}^* V_{ud} \left(z + \tau y\right)$$

where

$$z = c_1 - c_2$$
$$y = c_2 - c_3$$
$$\tau = -\frac{V_{ts}^* V_{td}}{V_{us}^* V_{ud}}$$

4-quark effective operators - $\Delta I = 3/2$

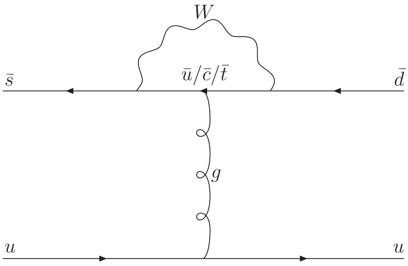
$$
\begin{aligned}
\mathcal{O}_{(27,1)} &= \quad (\bar{s}_i d_i)_L((\bar{u}_j u_j)_L - (\bar{d}_j d_j)_L) + (\bar{s}_i u_i)_L(\bar{u}_j d_j)_L \\
\mathcal{O}_{(8,8)} &= \quad (\bar{s}_i d_i)_L((\bar{u}_j u_j)_R - (\bar{d}_j d_j)_R) + (\bar{s}_i u_i)_L(\bar{u}_j d_j)_R \\
\mathcal{O}_{(8,8)mx} &= \quad (\bar{s}_i d_j)_L((\bar{u}_j u_i)_R - (\bar{d}_j d_i)_R) + (\bar{s}_i u_j)_L(\bar{u}_j d_i)_R
\end{aligned}
$$

where

$$(\bar{q}_1 q_2)_{L/R}(\bar{q}_3 q_4)_{L/R} \equiv \left(\bar{q}_1 \gamma^\mu (1 \mp \gamma^5) q_2\right) \left(\bar{q}_1 \gamma_\mu (1 \mp \gamma^5) q_2\right)$$

Operators are labelled by (L,R), which labels the dimension of the irreducible representation of chiral symmetry group $SU(3)_L \times SU(3)_R$ under which the operator transforms.

$\Delta I = 1/2$ - additional operators



- New operator - QCD penguin
- Effective operators have the form $(\bar{s}_i d_{i/j})_L \sum_a (\bar{q}_{a,j/i} q_{a,j})_{L/R}$
- They are isospin doublets - only contribute to $\Delta I = 1/2$
- (8,1) representation of chiral symmetry
- With 3 previously discussed operators we have 7 operators in total

Lattice vs continuum

- Finite volume effects

$$\langle \pi\pi \,|\, O_i \,|\, K \rangle_\infty = F \langle \pi\pi \,|\, O_i \,|\, K \rangle_{FV}$$

- Renormalisation
  Wilson coefficients must be in the same scheme as matrix elements (e.g. $\overline{\mathrm{MS}}$), so we're looking for conversion matrices:

$$M_i^{\overline{\mathrm{MS}}} = Z_{ij}^{LAT \to \overline{\mathrm{MS}}} M_j$$

- Discretisation effects
  Can be eliminated by taking continuum extrapolation through data points corresponding to ensembles with different lattice spacings.

Putting it all together...

The 'master equation':

$$A_2 = F\, V_{us}^* V_{ud} \frac{G_F}{\sqrt{2}} \sum_{i,j} \left(z_i^{\overline{\mathrm{MS}}}(\mu) + \tau y_i^{\overline{\mathrm{MS}}}(\mu) \right) Z_{ij}^{LAT \to \overline{\mathrm{MS}}}(a\mu)\, M_j^{LAT}(a)$$

with

$$M_i^{LAT}(a) = \langle \pi\pi \,|\, O_i \,|\, K \rangle_{FV}$$

Matrix elements

$$C_{K\to(\pi\pi)_I}(t) = \langle \sigma^\dagger_{\pi\pi;I}(t_{\pi\pi})\mathcal{O}_i(t)\sigma_K(0)\rangle$$
$$= \mathrm{Tr}\left(e^{-H(T-t_{\pi\pi})}\sigma^\dagger_{\pi\pi;I}e^{-H(t_{\pi\pi}-t)}\mathcal{O}_i e^{-Ht}\sigma_K(0)\right)$$

Insert complete set of states:

$$C_{K\to(\pi\pi)_I}(t) = \sum_{a,b,c}\langle a\,|\sigma^\dagger_{\pi\pi;I}\,|b\rangle\langle b\,|\mathcal{O}_i\,|c\rangle\langle c\,|\sigma_K(0)\,|a\rangle$$
$$\times\, e^{-E_a(T-t_{\pi\pi})}e^{-E_b(t_{\pi\pi}-t)}e^{-E_c t}$$
$$\xrightarrow[t,t_{\pi\pi}-t,T-t_{\pi\pi}\to\infty]{T-t_{\pi\pi}>>t,t_{\pi\pi}-t} \sum_{b,c}\langle 0\,|\sigma^\dagger_{\pi\pi;I}\,|b\rangle\langle b\,|\mathcal{O}_i\,|c\rangle\langle c\,|\sigma_K(0)\,|0\rangle$$
$$\times\, e^{-E_b(t_{\pi\pi}-t)}e^{-E_c t}$$

The lowest energy state c_0 corresponds to kaon at rest. However, b_0 corresponds to two pions at rest, which is not what we want.

Boundary conditions

We can control the allowed momenta of particles by choosing appropriate boundary conditions. E.g. using twisted boundary conditions in 1D:

$$\phi(0) = e^{i\theta}\phi(L)$$

Fourier transforming both sides gives the set of allowed momenta:

$$p = \frac{2\pi n - \theta}{L}$$

For $K \to \pi\pi$ with kaon at rest we are interested in:

- Periodic: $\theta = 0$, $p_{min} = 0$
- Antiperiodic: $\theta = \pi$, $p_{min} = \pm\pi/L$

Wigner-Eckart theorem

Choosing periodic boundary conditions for s and u quarks and antiperiodic for d quarks gives kaon at rest, π^+ and π^- with non-zero momentum, but π^0 at rest. Solution

Theorem (Wigner-Eckart)

$\langle JM \mid O^{j_1}_{m_1} \mid j_2 m_2 \rangle = \langle JM \mid j_1 m_1; j_2 m_2 \rangle \langle J \parallel O^{j_1} \parallel j_2 \rangle$

In I=2 case we have:

$$\langle (\pi\pi)^{I=2}_{I_3=0} \mid \mathcal{O}^{3/2}_{1/2} \mid K^0 \rangle = \frac{1}{\sqrt{2}} \langle \pi^+\pi^+ \mid \mathcal{O}^{3/2}_{3/2} \mid K^+ \rangle$$
$$= \sqrt{\frac{3}{2}} \langle \pi^+\pi^+ \mid \mathcal{O}'^{3/2}_{3/2} \mid K^+ \rangle$$

This choice of boundary conditions breaks isospin, but $|\pi^+\pi^+\rangle$ can only belong to I=2 representation (charge conservation).

$\Delta I = 3/2$ operators

The $\Delta I = 3/2$ operators now become:

$$\mathcal{O}^{(27,1)} = (\bar{s}_i u_i)_L \left(\bar{d}_j u_j\right)_L$$
$$\mathcal{O}^{(8,8)} = (\bar{s}_i u_i)_L \left(\bar{d}_j u_j\right)_R$$
$$\mathcal{O}^{(8,8)mx} = (\bar{s}_i u_j)_L \left(\bar{d}_j u_i\right)_R$$

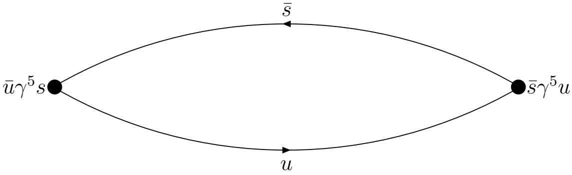
Three point correlation function

Using the above, the three-point correlation function becomes:

$$C_i^{K\to\pi\pi}(t) \approx \underbrace{\langle 0 \,|\sigma^\dagger_{\pi\pi}\,|\pi\pi\rangle}_{N_{\pi\pi}} \underbrace{\langle \pi\pi \,|\mathcal{O}_i\,| K\rangle}_{M_i} \underbrace{\langle K \,|\sigma_K\,|0\rangle}_{N_K} \times e^{-E_{\pi\pi}(t_{\pi\pi}-t)} e^{-m_K t}$$

Kaon correlation function

$$C_K(t) = \langle \sigma_K^\dagger(t)\sigma_K(0)\rangle \xrightarrow[t\to\infty]{} N_K^2 \left(e^{-m_K t} + e^{-m_K(T-t)}\right)$$

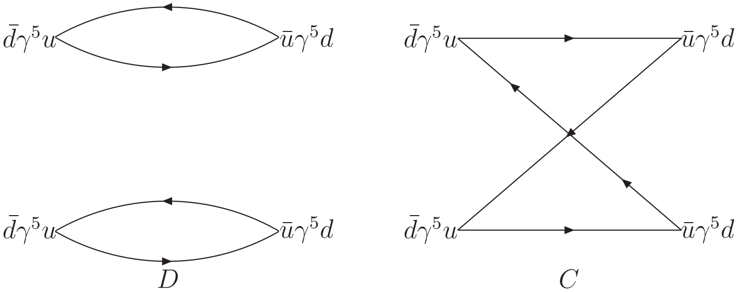


$$C_K(t) = -\mathrm{Tr}\left(S_d(t,0)S_s^\dagger(t,0)\right)$$

$$C_i^{K\to\pi\pi}(t) \approx N_{\pi\pi} M_i N_K e^{-E_{\pi\pi}(t_{\pi\pi}-t)} e^{-m_K t}$$

Two pion correlation function

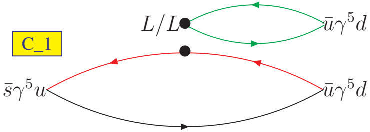
$$C_{\pi\pi} = \langle \sigma^\dagger_{\pi\pi}(t)\sigma_{\pi\pi}(0)\rangle$$
$$= |N_{\pi\pi}|^2 \left(e^{-E_{\pi\pi}t} + e^{-E_{\pi\pi}(T-t)} + C \right)$$



$$C_{\pi\pi}(t) = 2D(t) - 2C(t)$$

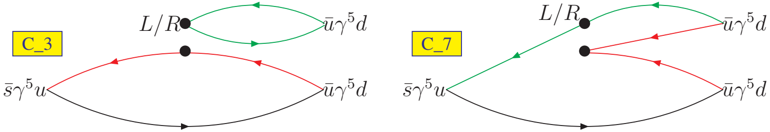
$$C_i^{K\to\pi\pi}(t) \approx N_{\pi\pi} M_i N_K e^{-E_{\pi\pi}(t_{\pi\pi}-t)} e^{-m_K t}$$

$K \to \pi\pi$ (27, 1) correlation function



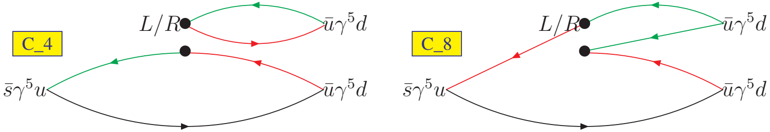
$$C^{K\to\pi\pi}_{(27,1)} = 2\left(C_1 + C_2\right)$$

$K \to \pi\pi$ (8, 8) correlation function



$$C_{(8,8)}^{K\to\pi\pi} = 2\left(C_3 - C_7\right)$$

$K \to \pi\pi$ $(8,8)mx$correlation function



$$C^{K\to\pi\pi}_{(8,8)mx} = 2\,(C_4 - C_8)$$

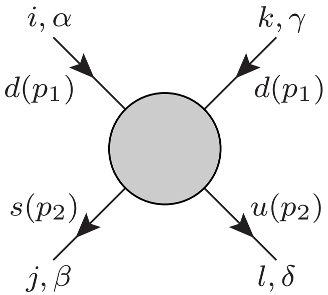
$$C_i^{K\to\pi\pi}(t) \approx N_{\pi\pi} M_i N_K e^{-E_{\pi\pi}(t_{\pi\pi}-t)} e^{-m_K t}$$

$$A_2 = F V_{us}^* V_{ud} \frac{G_F}{\sqrt{2}} \sum_{i,j} \left(z_i^{\overline{\mathrm{MS}}}(\mu) + \tau y_i^{\overline{\mathrm{MS}}}(\mu) \right) Z_{ij}^{LAT\to\overline{\mathrm{MS}}}(a\mu) M_j^{LAT}(a)$$

Nonperturbative renormalization

Need to convert to lattice quantities to $\overline{\mathrm{MS}}$ to use Wilson coefficients.

$$\begin{array}{ccccc} \textit{lattice} & \rightarrow & RI - SMOM & \rightarrow & \overline{\mathrm{MS}} \\ a & & \mu & & d = 4 - 2\epsilon \end{array}$$



$$p_1^2 = p_2^2 = (p_1 - p_2)^2 = \mu^2$$

We choose $\mu = 3$ GeV for $\Delta I = 3/2$ and $\mu = 1.53$ GeV in $\Delta I = 1/2$ calculation.

Converting lattice matrix elements to RI-SMOM

RI-SMOM defined by

$$\mathrm{Tr} P \Lambda_i |_{sym} = \mathrm{Tr} P \Lambda_{i,tree} |_{sym}$$

Λ_i is the amputated four-point Green function and P is a projection operator, which we choose to be:

$$\left[P^{(\not q)} \right]^{JI;LK}_{\beta\alpha;\delta\gamma} = \begin{pmatrix} \left[(\not q)_{\beta\alpha} (\not q)_{\delta\gamma} + (\not q \gamma^5)_{\beta\alpha} (\not q \gamma^5)_{\delta\gamma} \right] \delta^{JI} \delta^{LK} \\ \left[(\not q)_{\beta\alpha} (\not q)_{\delta\gamma} - (\not q \gamma^5)_{\beta\alpha} (\not q \gamma^5)_{\delta\gamma} \right] \delta^{JI} \delta^{LK} \\ \left[(\not q)_{\beta\gamma} (\not q)_{\delta\alpha} - (\not q \gamma^5)_{\beta\gamma} (\not q \gamma^5)_{\delta\alpha} \right] \delta^{JK} \delta^{LI} \end{pmatrix} .$$

Operators renormalize as:

$$Z_{ij}(\mu a) O_j^{LAT}(a) = O_i^{RI}(\mu)$$

So Λ will renormalize as:

$$\frac{Z_{ij}(\mu a)}{Z_q^2} \Lambda_j^{LAT}(a) = \Lambda_i^{RI}(\mu)$$

Calculating Z_q

Z_q is the quark renormalization constant which can be computed from vector current operator in a similar way:

$$\mathrm{Tr} P' \Lambda_V^\mu \Big|_{sym} = \mathrm{Tr} P' \Lambda_{V,tree}^\mu \Big|_{sym}$$

where Λ_V is two-point amputated Green function, which renormalizes as:

$$\frac{Z_V(\mu a)}{Z_q^2} \Lambda_V^{LAT,\mu}(a) = \Lambda_V^{RI,\mu}(\mu)$$

For our choice of projector:

$$\frac{Z_q^{(\not q)}}{Z_V} = \frac{q^\mu}{12 q^2} \mathrm{Tr} \Lambda_V^\mu \not q, \quad \text{and}$$

Finally, Z_V can be calculated using:

$$2 m_\pi = Z_V \langle \pi, 0 \mid V^4 \mid \pi, 0 \rangle$$

This can be used to calculate Z_{ij}

Some notes on chiral symmetry

- We use domain wall fermion action, which preserves (approximately) chiral symmetry.
- Only operators in the same irreducible representation of $SU(3)_L \times SU(3)_R$ flavour symmetry can mix with each other under renormalization (Z_{ij} is block diagonal)
- (27,1) operator renormalizes multiplicatively.
- (8,8) operators mix with each other.
- (in the $\Delta I = 1/2$ case) (8,1) operators mix with each other.
- The authors of (1505.05289) argue that $SU(3)_V$ symmetry is sufficient to reproduce the operator mixing pattern and it is therefore possible to use non-chiral formulation of fermions (e.g. Wilson).
- However, without chiral symmetry we introduce O(a) effects.

$$A_2 = F V_{us}^* V_{ud} \frac{G_F}{\sqrt{2}} \sum_{i,j} \left(z_i^{\overline{\mathrm{MS}}}(\mu) + \tau y_i^{\overline{\mathrm{MS}}}(\mu) \right) Z_{ij}^{LAT \to \overline{\mathrm{MS}}}(a\mu) M_j^{LAT}(a)$$

Finite volume effects - Lellouch-Lüscher formula

FV correction is given by the Lellouch-Lüscher factor:

$$F^2 = 8\pi V^2 \left(q\frac{\partial\phi}{\partial q} + p\frac{\partial\delta}{\partial p} \right) \frac{m_K E_{\pi\pi}^2}{p^3}$$

with ϕ and δ given by Lüscher quantization condition:

$$\delta(p) + \phi(q)\big|_{q=\frac{pL}{2\pi}} = n\pi$$

- δ is the 2-pion s-wave (I=0) phase shift
- ϕ is given by:

$$\tan\phi = \frac{q\pi^{3/2}}{Z_{00}(1;q)}$$

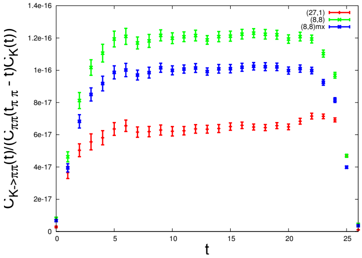
$$Z_{00}(1;q^2) = \frac{1}{4\pi^2}\sum_{n\in Z^3}\frac{1}{n^2-q^2}$$

Caveat: $\frac{\partial\delta}{\partial p}$ can't be calculated directly (requires approximation).

$$A_2 = F V_{us}^* V_{ud} \frac{G_F}{\sqrt{2}} \sum_{i,j} \left(z_i^{\overline{\mathrm{MS}}}(\mu) + \tau y_i^{\overline{\mathrm{MS}}}(\mu) \right) Z_{ij}^{LAT \to \overline{\mathrm{MS}}}(a\mu) M_j^{LAT}(a)$$

Summary of results

[MeV]	$48^3 \times 96 \times 24$	$64^3 \times 128 \times 12$	$32^3 \times 64 \times 12$ (G)
m_π	139.1(2)	139.2(3)	143.1(20)
m_K	498.82(26)	507.4(4)	490.6(2.6)
$E_{\pi\pi}$	496.5(16)	507.0(16)	498(11)
$m_K - E_{\pi\pi}$	2.4(24)	2.1(26)	7(11)



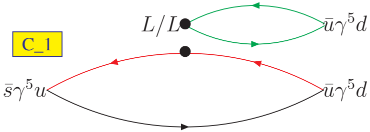
Ratios of 3-point to 2-point correlation functions which removes the time dependence. We see clear plateaux.

A_2 cancellation

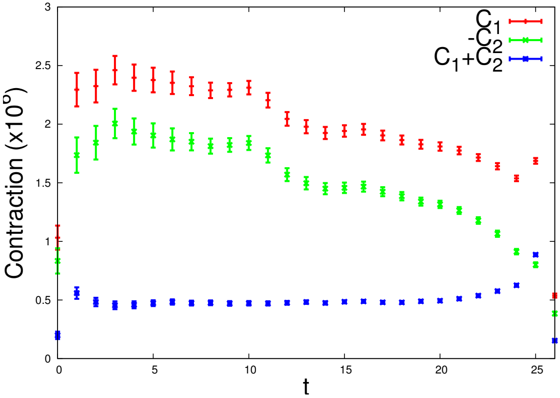
For I=2 channel the real parts of Wilson coefficients corresponding to (8,8) operators are small. The following is a good approximation:

$$\mathrm{Re}(A_2) \propto \mathcal{O}^{(27,1)}$$

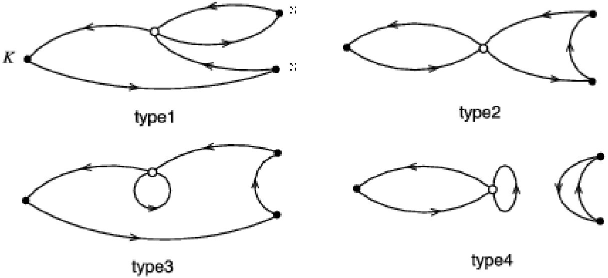
which in turn is equal to sum of the following two diagrams:



A_2 cancellation



$\Delta I = 1/2$ - additional contractions



- Only 'type 1' present in $\Delta I = 3/2$
- 8 contraction of types 1 and 2 each and 16 of type 3 and 4 each = 48 contractions in total.

$\Delta I = 1/2$ - G-parity

We're looking for boundary conditions such that all pions are antiperiodic and kaon is periodic. This can be achieved using G-parity boundary conditions:

$$u \to C\bar{d}^T$$
$$d \to -C\bar{u}^T$$

- All pions antiperiodic.
- K is not a G-parity eigenstate $\to$ introduce a fictitous s' quark - a G-parity partner of s.
- Can construct a G-parity even kaon state - the unphysical contribution is suppressed by the volume.
- Additional s' quarks can be eliminated from the sea by taking a square root of the strange quark determinant.

$\Delta I = 1/2$ - vacuum subtraction

The $|\, I = 0\rangle$ state has the same quantum numbers as the vacuum state $|\,0\rangle$. To get the $|\, I = 0\rangle$ state from the correlation function, we need to subtract the vacuum contribution. For example:

$$
\begin{aligned}
C_{K\to\pi\pi;i}(t) &\equiv \langle O^{\dagger}_{\pi\pi;I=0}(t_{\pi\pi}) Q_i(t) O_K(0)\rangle \\
&= \langle 0\,|\, O_{\pi\pi;I=0}\,|0\rangle\langle 0\,|\, Q_i(t) O_K(0)\,|0\rangle + \ldots
\end{aligned}
$$

This subtraction can be done contraction-by-contraction.

$\Delta I = 1/2$ - operator subtraction

Among new contractions, type 3 and type 4 contain the loop on the operator. This loop contains a quadratic divergence, which can be absorbed into a counterterm of the form $\bar{s}\gamma^5 d$.
Adding this counterterm is *not* required in the continuum:

$$i(m_s + m_d)\bar{s}\gamma^5 d = \partial_\mu \left(\bar{s}\gamma^\mu\gamma^5 d\right)$$

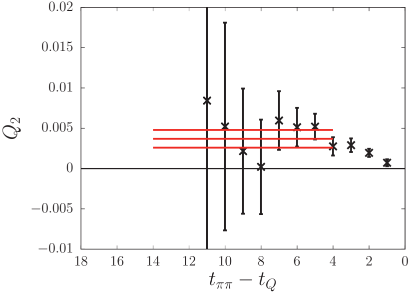
$$\langle f \mid \partial^\mu O(x) \mid i \rangle = (p_f - p_i)^\mu \langle f \mid O \mid i \rangle e^{i(p_f - p_i)x}$$

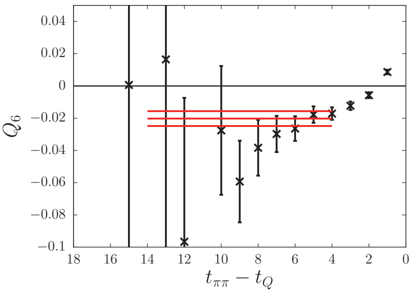
Vanishes for physical matrix elements ($p_i = p_f$).
On the lattice we need to tune the energies by hand, so in general $p_i \neq p_f$.
We can remove the quadratic divergence using the renormalisation prescription:

$$Q_i^R = Q_i - \alpha_i \bar{s}\gamma^5 d$$

$$\langle 0 \mid Q_i^R \mid K \rangle = 0$$

Results - A_0


0.02
0.015
0.01
0.005
0
−0.005
−0.01
Q_2
18 16 14 12 10 8 6 4 2 0
$t_{\pi\pi} - t_Q$


0.04
0.02
0
−0.02
−0.04
−0.06
−0.08
−0.1
Q_6
18 16 14 12 10 8 6 4 2 0
$t_{\pi\pi} - t_Q$

Final results

	Re(A_2)	Im(A_2)
48^3	$1.386(12) \times 10^{-8}$	$-6.174(49) \times 10^{-13}$
64^3	$1.4386(95) \times 10^{-8}$	$-6.548(78) \times 10^{-13}$
continuum	$1.50(4)(14) \times 10^{-8}$	$-6.99(20)(84) \times 10^{-13}$

C.f. experimental values:

- $\mathrm{Re} A_2 = 1.4787(31) \times 10^{-8}$ GeV from charged kaon decays
- $\mathrm{Re} A_2 = 1.570(53) \times 10^{-8}$ GeV from neutral kaon decays

	Re(A_0)	Im(A_0)
32^3	$4.66(1.00)(1.26) \times 10^{-7}$	$1.90(1.23)(1.08) \times 10^{-11}$

C.f. experimental values:

- $\mathrm{Re} A_0 = 3.3201(18) \times 10^{-7}$ GeV

Final result for $\mathrm{Re}(\varepsilon'/\varepsilon) = 1.38(5.15)(4.59) \times 10^{-4}$ which is 2.1σ below the experimental value of $\mathrm{Re}(\varepsilon'/\varepsilon) = 16.6(2.3) \times 10^{-4}$.

What next?

Dominant contributions to the error budget:

contribution	$\Delta I = 3/2$	$\Delta I = 1/2$
Finite lattice spacing	included in statistical error	12%
Wilson coefficients	6.8%	12%
Renormalisation	2.8%	15%
Derivative of the phase shift	1.6%	11%

Improvement of the renormalisation constant calculation already underway - see Chris Kelly's Lattice 2016 talk.

Conclusions

- $K \to \pi\pi$ decays important for two reasons: ε'/ε and $\Delta I = 1/2$ rule
- very challenging at physical kinematics - difficult to have pions with the right momentum
- we find $\varepsilon'/\varepsilon = 1.38(5.15)(4.59) \times 10^{-4}$, which is 2.1σ below the experimental value
- we determined the origin of the $\Delta I = 1/2$ rule, which is the cancellation between matrix elements of operators Q_1 and Q_2 in the $\Delta I = 3/2$ channel
- first and currently only calculation of this process at physical kinematics; the only other threshold ($m_K = 2m_\pi$) calculation is (1505.05289)

Thank you for your attention!